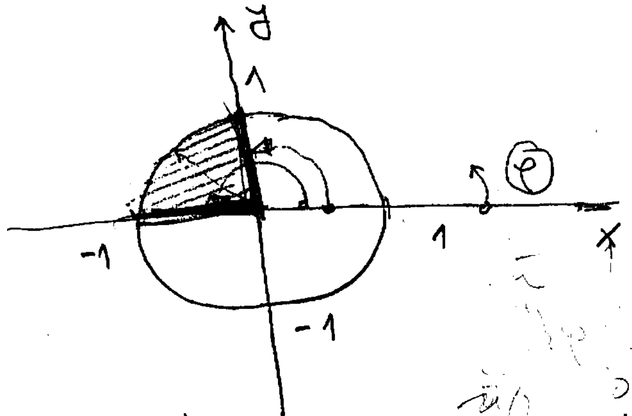
6.4.

$$x = \rho\cos\varphi \,,\; y = \rho\sin\varphi$$

(1.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \wedge x \leq 0 \wedge y \geq 0 \}$

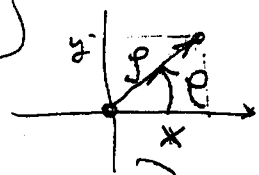


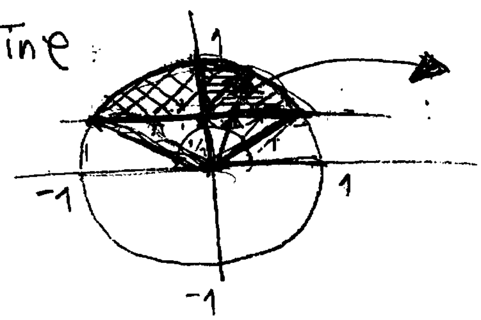
$$\begin{cases} \frac{\pi}{2} \leq \varphi \leq \pi \\ 0 \leq \rho \leq 1 \end{cases}$$

(2.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1 \wedge y \geq \frac{1}{2} \}$

$\rho^2 \leq 1$ $\quad$ $\rho\sin\varphi \geq \frac{1}{2}$

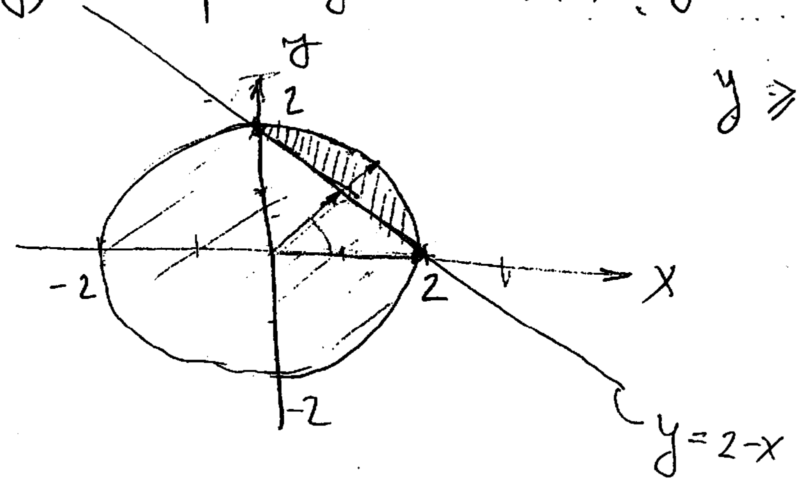
$\rho \geq \frac{1}{2\sin\varphi}$





$$\begin{cases} \frac{1}{2\sin\varphi} \leq \rho \leq 1 \\ 0 \leq \varphi \leq \pi \end{cases}$$

(3.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4 \wedge x + y - 2 \geq 0 \}$



$y \geq 2 - x$

$x + y \geq 2$

$\rho\cos\varphi + \rho\sin\varphi \geq 2$

$\rho(\cos\varphi + \sin\varphi) \geq 2$

$\boxed{\rho \geq \dfrac{2}{\cos\varphi + \sin\varphi}}$

$\rho^2\cos\varphi^2 + \rho^2\sin^2\varphi \leq 4$

$\rho^2(\cos\varphi^2 + \sin^2\varphi) \leq 4$

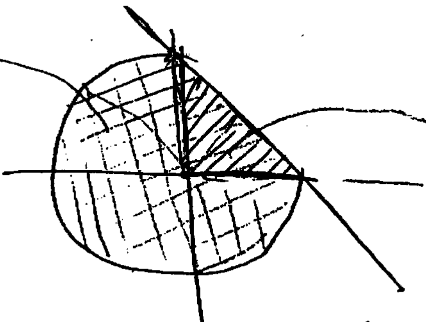
$\rho^2 \leq 4$

$\boxed{\rho \leq \pm 2}$

$$\begin{cases} \dfrac{2}{\cos\varphi+\sin\varphi} \leq \rho \leq 2 \\ 0 \leq \varphi \leq \dfrac{\pi}{2} \end{cases}$$

4.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 4 \wedge x + y - 2 \leq 0 \}$

$$\begin{cases} 0 \leq \rho \leq 2 \\ \dfrac{\pi}{2} \leq \varphi \leq 2\pi \end{cases}$$



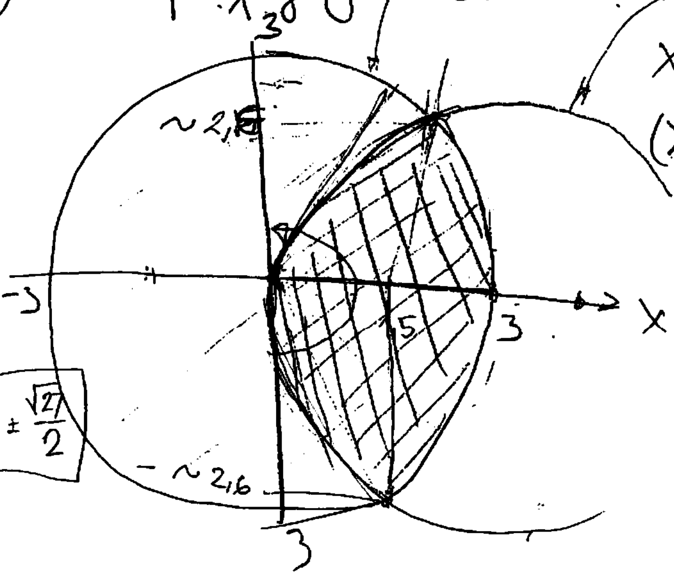
$$\begin{cases} 0 \leq \rho \leq \dfrac{2}{\cos\varphi+\sin\varphi} \\ 0 \leq \varphi \leq \dfrac{\pi}{2} \end{cases}$$

$4 + 4 \leq 12 \quad 16 \leq 24$

5.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 9 \wedge x^2 + y^2 \leq 6x \}$

$x^2 + y^2 - 6x \leq 0$

$(x-3)^2 + y^2 \leq 9$



$\frac{9}{4} + y^2 = 9$

$y^2 = 9 - \frac{9}{4}$

$y^2 = \frac{36-9}{4} = \frac{27}{4}$

$\boxed{y = \pm\frac{\sqrt{27}}{2}}$

$9 = 6x$

$\boxed{x = \frac{9}{6} = \frac{3}{2}}$

$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 9 \quad \wedge \quad \rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 6\rho\cos\varphi$

$\rho^2 = 9 \quad \wedge \quad \rho^2 = 6\rho\cos\varphi$

$\boxed{\rho = 3}$ $\quad \cancel{9 = 6\rho\cos\varphi}$ $\quad \boxed{\rho = 6\cos\varphi}$

$\cancel{\rho = \frac{9}{6\cos\varphi}} \rightarrow \boxed{\cancel{\rho = \frac{3}{2\cos\varphi}}}$

$$\left\{ \begin{array}{l} 6\cos\varphi \le \rho \le 3 \\ -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2} \end{array} \right\}$$

(6) $D = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \ge 9 \wedge x^2 + y^2 \le 6y\}$

$x^2 + (y-3)^2 \le |9|$

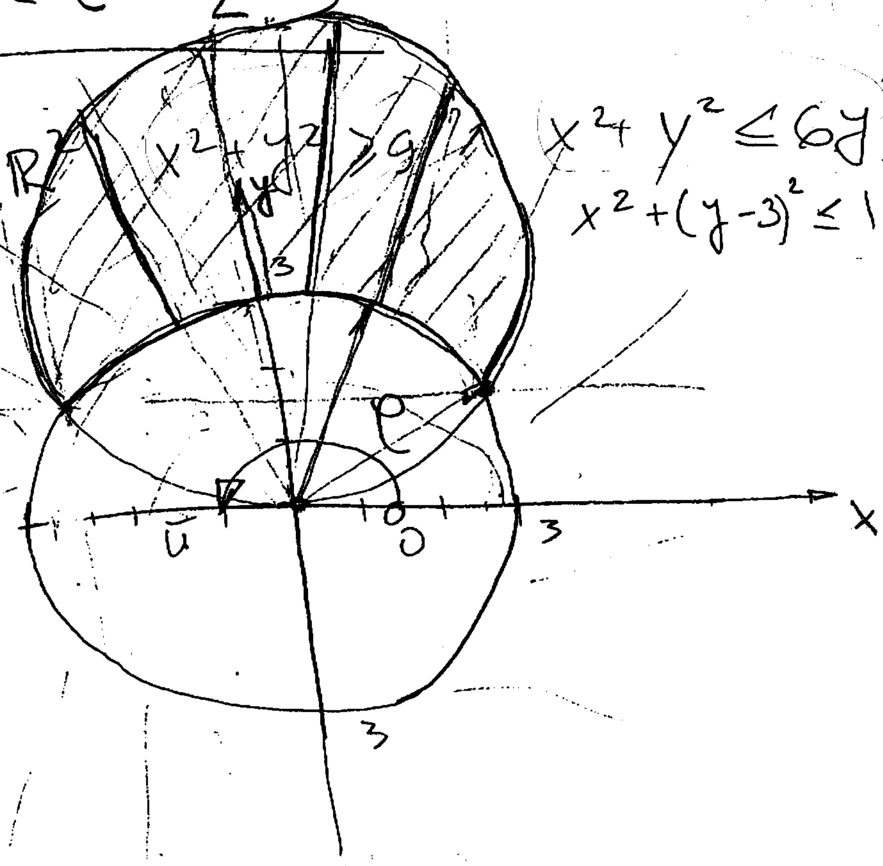
$9 = 6y$

$\boxed{y = \frac{3}{2}}$

$x^2 + \frac{9}{4} = 9$

$x^2 = \frac{36-9}{4} = \frac{27}{4}$

$\boxed{x = \pm\frac{\sqrt{27}}{2}}$



$\rho^2 \ge 9 \quad \wedge \quad \rho^2 = 6\rho\sin\varphi$

$\boxed{\rho = 3}$ $\quad \boxed{\rho = 6\sin\varphi}$

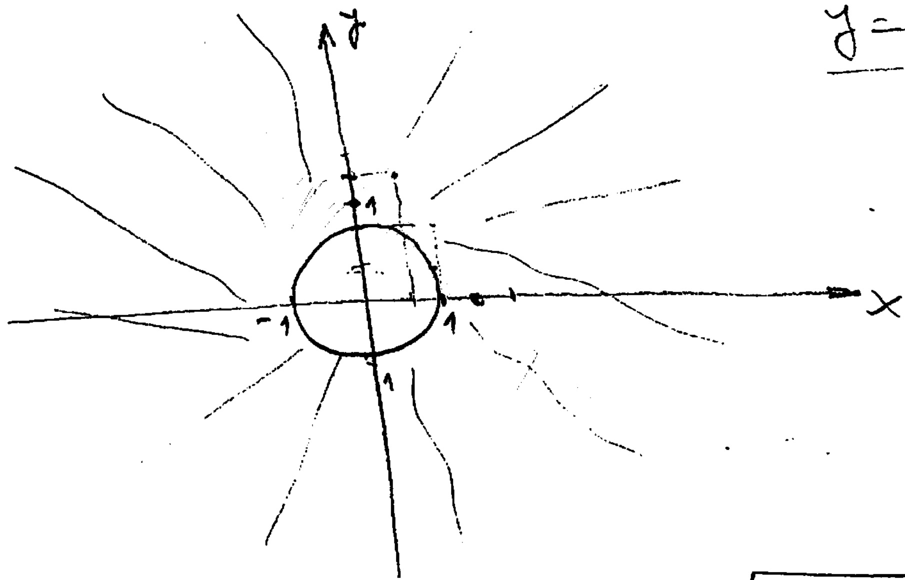
$$\left\{ \begin{array}{l} 3 \le \rho \le 6\sin\varphi \\ 0 \le \varphi \le \pi \end{array} \right\}$$

7. $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1 \wedge \underline{x+y \leq \sqrt{2}}\}$

$\downarrow$

$x^2+2xy+y^2 \leq 2$

$y = \sqrt{2} - x$



$x^2 + (\sqrt{2}-x)^2 = 1$

$x^2 + 2 - 2\sqrt{2}x + x^2 = 1$

$2x^2 - 2\sqrt{2}x + 2 = 1 \quad / \cdot 2$

$x^2 - \sqrt{2}x + 1 = \frac{1}{2}$

$x^2 + \sqrt{2}x + \frac{1}{2} = 0$

$x_{1,2} = \frac{-\sqrt{2} \pm \sqrt{2-2}}{2}$

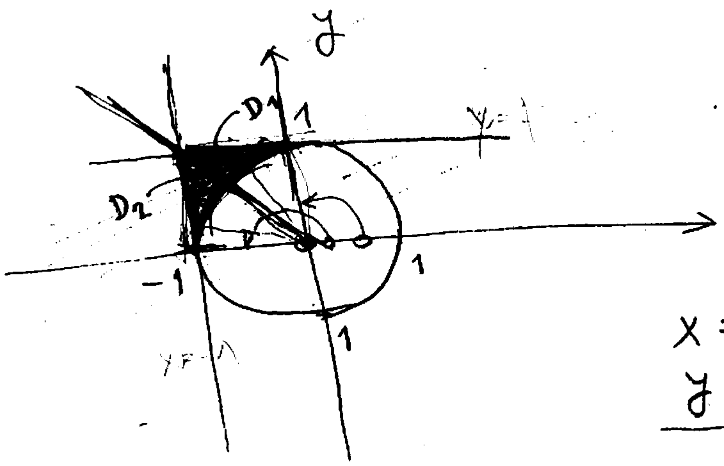
$\boxed{x_{1,2} = -\frac{\sqrt{2}}{2}}$ $\sim 0{,}707$

$y_{1,2} = \sqrt{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}+\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$

$\sim 2{,}12$

$1 < \rho <$

8. $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1 \wedge -1 \leq x \leq 0 \wedge 0 \leq y \leq 1\}$



$135^\circ = \frac{3\pi}{4}$

$x = \rho\cos\varphi$
$y = \rho\sin\varphi$

$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 1$

$\rho^2 = 1 \quad \rho = \pm 1 \Rightarrow \boxed{\rho = 1}$

$-1 = \rho\cos\varphi$

$\boxed{\rho = -\frac{1}{\cos\varphi}}$

$1 = \rho\sin\varphi$

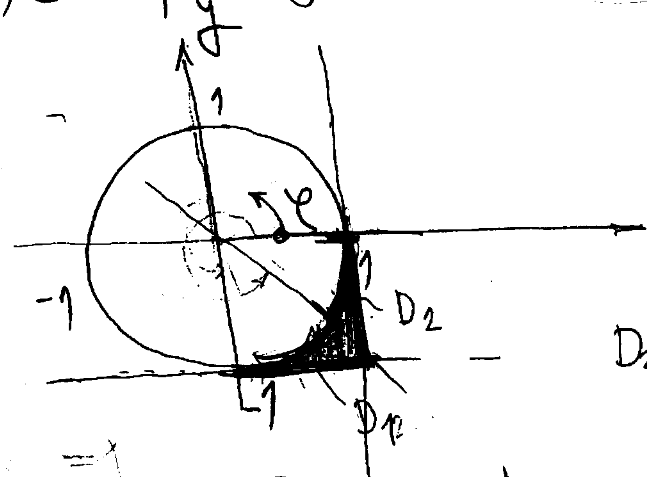
$\boxed{\rho = \frac{1}{\sin\varphi}}$

$D_1 = \begin{cases} 1 \leq \rho \leq \frac{1}{\sin\varphi} \\ \frac{\pi}{2} \leq \varphi \leq \frac{3\pi}{4} \end{cases}$

$D_2 = \begin{cases} 1 \leq \rho \leq -\frac{1}{\cos\varphi} \\ \frac{3\pi}{4} \leq \varphi \leq \pi \end{cases}$

9. $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1 \wedge 0 \leq x \leq 1 \wedge -1 \leq y \leq 0\}$

$\frac{3\pi}{4} + \pi = \frac{7\pi}{4}$

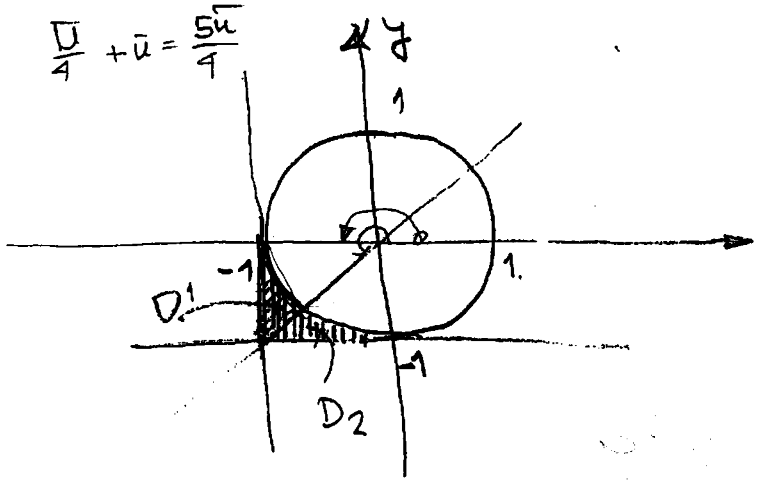


$D_1 : \begin{cases} 1 \leq \rho \leq -\frac{1}{\sin\varphi} \\ \frac{3\pi}{2} \leq \varphi \leq \frac{7\pi}{4} \end{cases}$

$D_2 : \begin{cases} 1 \leq \rho \leq \frac{1}{\cos\varphi} \\ \frac{7\pi}{4} \leq \rho \leq 2\pi \end{cases}$

(10) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2 \geq 1 \wedge -1 \leq x \leq 0 \wedge -1 \leq y \leq 0 \}$

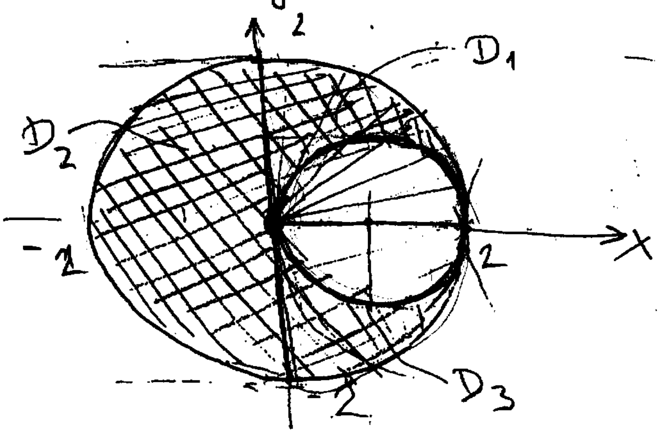
$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$



$D_1 : \left\{ \begin{array}{l} 1 \leq \rho \leq -\frac{1}{\cos\varphi} \\ \pi \leq \varphi \leq \frac{5\pi}{4} \end{array} \right\}$

$D_2 : \left\{ \begin{array}{l} 1 \leq \rho \leq -\frac{1}{\sin\varphi} \\ \frac{5\pi}{4} \leq \varphi \leq \frac{3\pi}{2} \end{array} \right.$

(11.) $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 4 \wedge x^2+y^2 \geq 2x \}$



$x^2+y^2-2x \geq 0$

$(x-1)^2+y^2-1 \geq 0$

$(x-1)^2+y^2 \geq 1$

$-2x+1$?

$4 = 2x \rightarrow \boxed{x=2}, \boxed{y=0}$

$\rho^2 \leq 4 \quad \boxed{\rho \leq \pm 2}$

$\cancel{\rho^2} \geq 2\cancel{\rho}\cos\varphi \quad \boxed{\rho \geq 2\cos\varphi}$

$D_1 : \left\{ \begin{array}{l} 2\cos\varphi \leq \rho \leq 2 \\ 0 \leq \varphi \leq \frac{\pi}{2} \end{array} \right\}$

$D_2 : \left\{ \begin{array}{l} 0 \leq \rho \leq 2 \\ \frac{\pi}{2} \leq \varphi \leq \frac{3\pi}{2} \end{array} \right\}$

$D_3 : \left\{ \begin{array}{l} 2\cos\varphi \leq \rho \leq 2 \\ \frac{3\pi}{2} \leq \varphi \leq 2\pi \end{array} \right\}$

12. $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le x \wedge x^2+y^2 \le y\}$

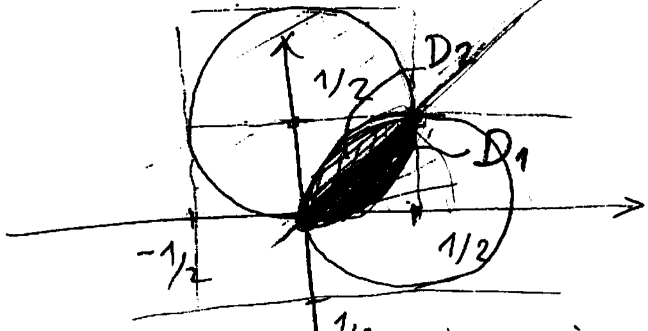
$x^2+y^2-x=0$

$(x-\frac{1}{2})^2+y^2-\frac{1}{4}=0$

$\boxed{(x-\frac{1}{2})^2+y^2=\frac{1}{4}}$

$x^2+y^2-y=0$

$\boxed{x^2+(y-\frac{1}{2})^2=\frac{1}{4}}$



$\rho^2 \le \rho\cos\varphi$

$\boxed{\rho \le \cos\varphi}$

$\boxed{\rho \le \sin\varphi}$

$D_1: \begin{cases} 0 \le \rho \le \sin\varphi \\ 0 \le \varphi \le \pi/4 \end{cases}$

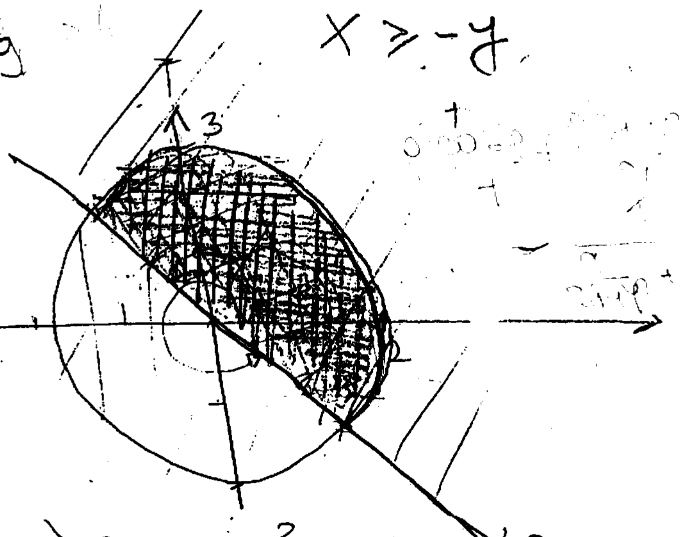
$D_2: \begin{cases} 0 \le \rho \le \cos\varphi \\ \pi/4 \le \varphi \le \pi/2 \end{cases}$

14. $D = \{(x,y) \in \mathbb{R}^2 \mid 9x^2+4y^2 \le 36 \quad x+y \ge 0\}$

$x \ge -y$

$9x^2+4y^2 \le 36 \;/ \frac{1}{36}$

$\boxed{\frac{x^2}{4}+\frac{y^2}{9} \le 1}$



$\cancel{9(\rho^2\cos^2\varphi)+4(\rho^2\sin^2\varphi)=36}$

$\cancel{9\rho^2\cos^2\varphi+4\rho^2\sin^2\varphi=36}$

$\cancel{\rho^2(9\cos^2\varphi+4\sin^2\varphi)=36}$

$\cancel{\rho^2=\frac{36}{9\cos^2\varphi+4\sin^2\varphi}}$ $\rightarrow$ $9-9\sin^2\varphi+4\sin^2\varphi$

$9-5\sin^2\varphi$

$\boxed{\rho = \frac{6}{\sqrt{9\cos^2\varphi+4\sin^2\varphi}}}$

$$\frac{\rho^2\cos^2\varphi}{4} + \frac{\rho^2\sin^2\varphi}{9} \le 1$$

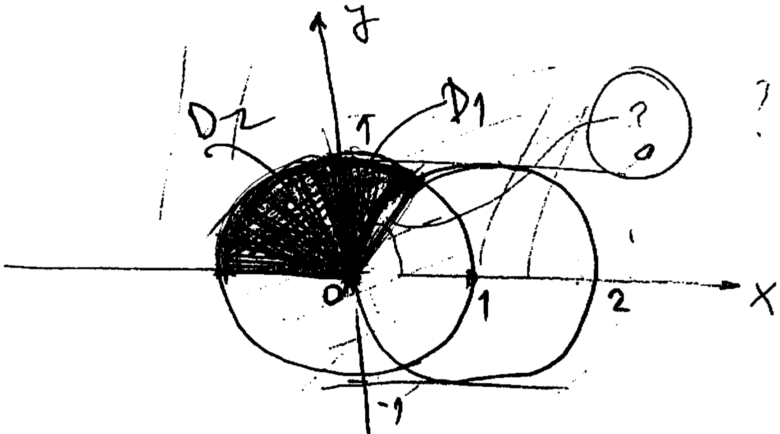
$$\rho\cos\varphi + \rho\sin\varphi \geq 0$$

$$\rho\,(\cos\varphi+\sin\varphi) \geq 0 \quad \boxed{\rho = 0}$$

$$D: \begin{cases} 0 \le \rho \le \dfrac{6}{\sqrt{9\cos^2\varphi + 4\sin^2\varphi}} \\ -\dfrac{3\pi}{4} \le \varphi \le \dfrac{3\pi}{4} \end{cases}$$

17. $D = \{ (x,y) \in \mathbb{R}^2 \mid x^2+y^2 \le 1 \wedge x^2+y^2 \geq 2x \wedge y \geq 0$

$$(x-1)^2 + y^2 \geq 1$$



$$D_1: \begin{cases} 2\cos\varphi \le \rho \le 1 \\ 0 \le \varphi \le \dfrac{\pi}{2} \end{cases}$$

$$D_2: \begin{cases} 0 \le \rho \le 1 \\ \dfrac{\pi}{2} \le \varphi \le \pi \end{cases}$$

$\rho^2 = 1 \quad \wedge \quad \rho^2 = 2\rho\cos\varphi \quad \wedge \quad \rho\sin\varphi \geq 0$

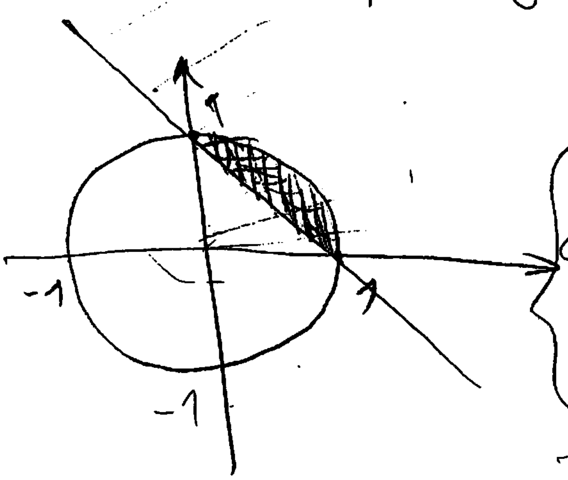
$\rho = 2\cos\varphi \qquad\qquad \rho = 0$

$\rho\cos\varphi + \rho\sin\varphi = 1$

$\boxed{\rho_1 = \dfrac{1}{\cos\varphi+\sin\varphi}}$

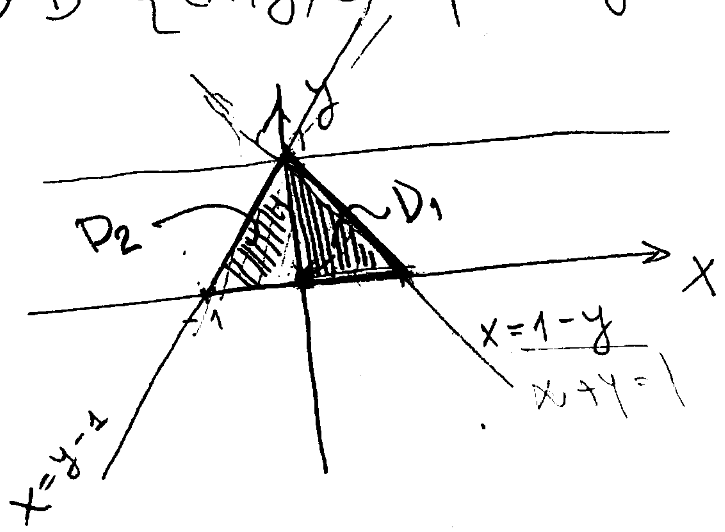
18. $D: \{ (x,y) \in \mathbb{R}^2 \mid x^2 + y^2 \le 1 \wedge x+y-1 \geq 0 \}$

$$y \geq 1 - x$$



$$\begin{cases} \dfrac{1}{\cos\varphi+\sin\varphi} \le \rho \le 1 \\ 0 \le \varphi \le \dfrac{\pi}{2} \end{cases}$$

19) $D=\{(x,y)\in\mathbb{R}^2 \mid 0\le y\le 1 \wedge y-1\le x\le 1-y\}$



$D_1:\begin{cases} 0\le\rho\le\dfrac{1}{\cos\varphi+\sin\varphi} \\ 0\le\varphi\le\dfrac{\pi}{2}\end{cases}$

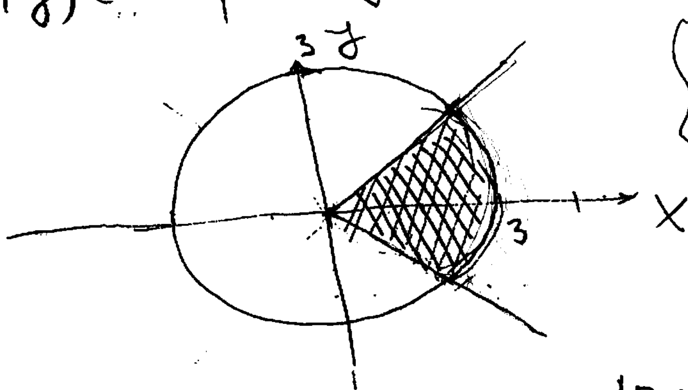
$D_2:\begin{cases} 0\le\rho\le-\dfrac{1}{\cos\varphi-\sin\varphi} \\ \dfrac{\pi}{2}\le\varphi\le\pi\end{cases}$

$\rho\cos\varphi+\rho\sin\varphi=1$

$\boxed{\rho=\dfrac{1}{\cos\varphi+\sin\varphi}}$

$\rho\cos\varphi-\rho\sin\varphi=-1$

20) $D=\{(x,y)\in\mathbb{R}^2 \mid x^2+y^2\le 9 \wedge x-|y|\ge 0\}$

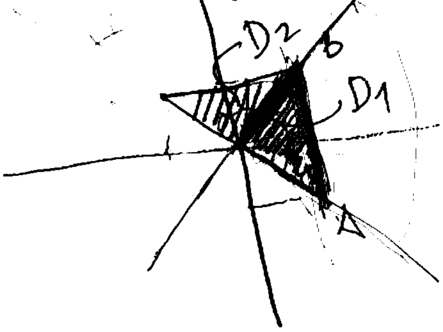


$\begin{cases} 0\le\rho\le 3 \\ -\dfrac{\pi}{4}\le\varphi\le\dfrac{\pi}{4}\end{cases}$

22) D: унутрашњости троугла ABC, $A(1,-1)$, $B(1,1)$, $C(-1,1)$

$x\ge 1$

$\rho\cos\varphi=1$



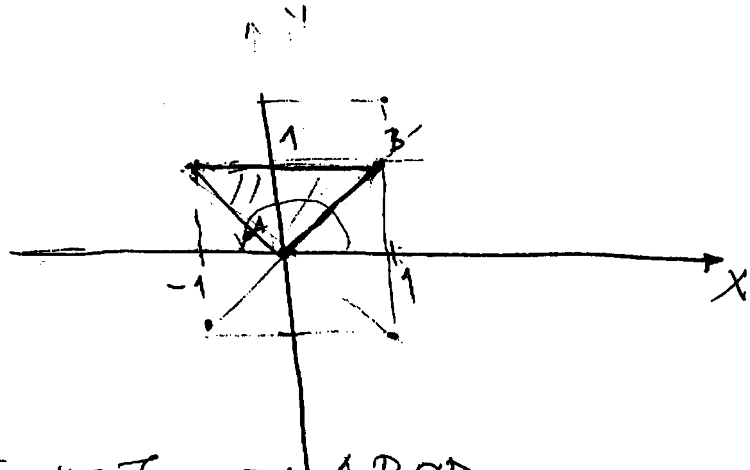
$D_1:\begin{cases} 0\le\rho\le\dfrac{1}{\cos\varphi} \\ -\dfrac{\pi}{4}\le\varphi\le\dfrac{\pi}{4}\end{cases}$

$D_2:\begin{cases} 0\le\rho\le\dfrac{1}{\sin\varphi} \\ \dfrac{\pi}{4}\le\varphi\le\dfrac{3\pi}{4}\end{cases}$

(24) D : унутрашност троугла ABC

$A(0,0),\ B(1,1),\ C(-1,1)$

$$\begin{cases} 0 \le \rho \le \dfrac{1}{\sin\varphi} \\ \dfrac{\pi}{4} \le \varphi \le \dfrac{3\pi}{4} \end{cases}$$

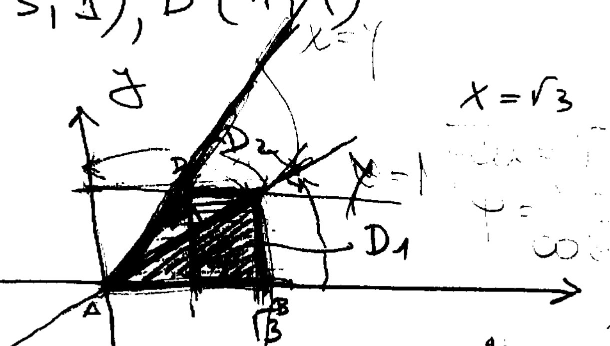


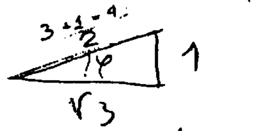
(26.) D : унутрашност четвороугла ABCD

$A(0,0),\ B(\sqrt{3},0),\ C(\sqrt{3},1),\ D(1,1)$

$$D_1 : \begin{cases} 0 \le \rho \le \dfrac{\sqrt{3}}{\cos\varphi} \\ 0 \le \varphi \le \dfrac{\pi}{6} \end{cases}$$

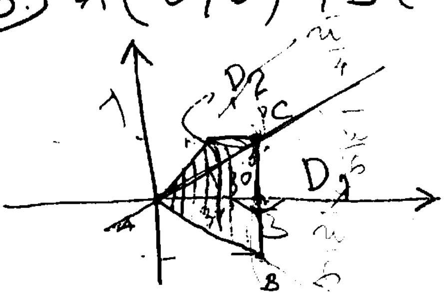
$$D_2 : \begin{cases} 0 \le \rho \le \dfrac{1}{\sin\varphi} \\ \dfrac{\pi}{6} \le \varphi \le \dfrac{\pi}{4} \end{cases}$$





$\operatorname{tg}\varphi = \dfrac{1}{\sqrt{3}}$

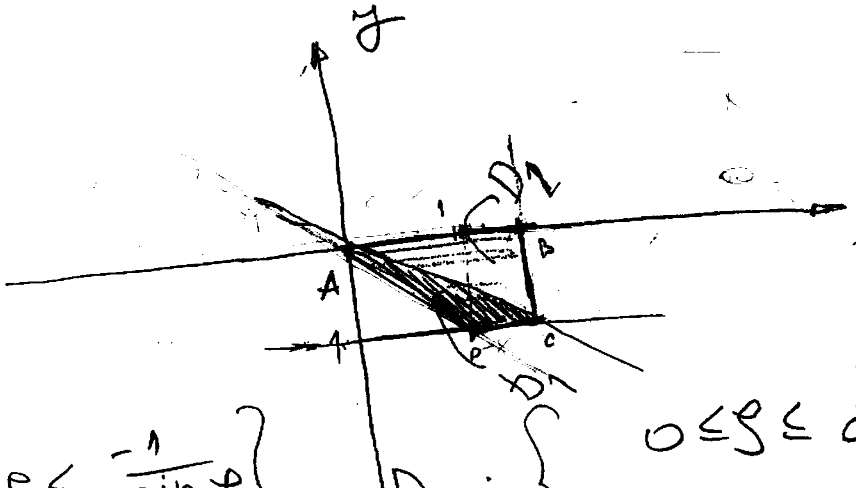
(28.) $A(0,0),\ B(\sqrt{3},-1),\ C(\sqrt{3},1),\ D(1,1)$



$$D_1 : \begin{cases} 0 \le \rho \le \dfrac{\sqrt{3}}{\cos\varphi} \\ -\dfrac{\pi}{6} \le \varphi \le \dfrac{\pi}{6} \end{cases}$$

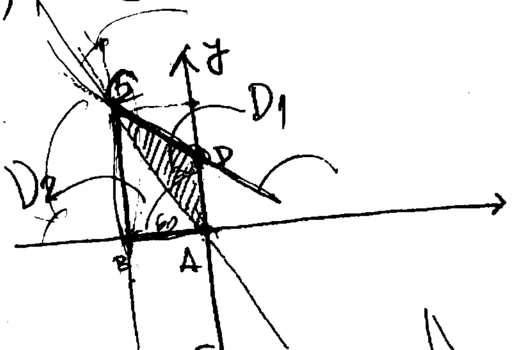
$$D_2 : \begin{cases} 0 \le \rho \le \dfrac{1}{\sin\varphi} \\ \dfrac{\pi}{6} \le \varphi \le \dfrac{\pi}{4} \end{cases}$$

30. D : $A(0,0)$, $B(\sqrt{3},0)$, $C(\sqrt{3},-1)$, $D(1,-1)$



$$D_1 : \begin{cases} 0 \le \rho \le \dfrac{-1}{\sin\varphi} \\ -\dfrac{\pi}{4} \le \varphi \le -\dfrac{\pi}{6} \end{cases} \qquad D_2 : \begin{cases} 0 \le \rho \le \dfrac{\sqrt{3}}{\cos\varphi} \\ -\dfrac{\pi}{6} \le \varphi \le 0\ (2\pi) \end{cases}$$

32. D : $A(0,0)$, $B(-1,0)$, $C(-1,\sqrt{3})$, $D(0,\sqrt{3}-1)$



$y = \alpha x + \beta$

$\sqrt{3} = -\alpha + \beta$

$\boxed{\sqrt{3}-1 = \beta}$

$\sqrt{3} = -\alpha + \sqrt{3} - 1$

$\cancel{\sqrt{3}} - \sqrt{3} + 1 = -\alpha$

$1 = -\alpha$

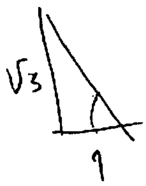
$\boxed{-1 = \alpha}$

$\boxed{y = -x + \sqrt{3} - 1}$

$\rho \sin\varphi = -\rho\cos\varphi + \sqrt{3} - 1$

$\rho(\sin\varphi + \cos\varphi) = \sqrt{3} - 1$

$\boxed{\rho = \dfrac{\sqrt{3}-1}{\sin\varphi + \cos\varphi}}$



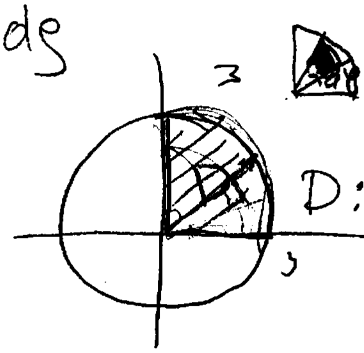
$$D_1 : \begin{cases} 0 \le \rho \le \dfrac{\sqrt{3}-1}{\sin\varphi + \cos\varphi} \\ \dfrac{\pi}{2} \le \varphi \le \dfrac{\pi}{6} \end{cases} \qquad D_2 : \begin{cases} 0 \le \rho \le -\dfrac{1}{\cos\varphi} \\ \dfrac{\pi}{6} \le \varphi \le \pi \end{cases}$$

(1.) $\iint_D \sqrt{x^2+y^2}\,dx\,dy$, D је четвртина круга у првом квадранту

$x^2+y^2=9$.

$x = \rho\cos\varphi$ $\quad d\varphi\, d\rho$

$y = \rho\sin\varphi$



$D: \begin{cases} 0 \le \rho \le 3 \\ 0 \le \varphi \le \frac{\pi}{2} \end{cases}$

$\iint_D \sqrt{\rho^2\cos^2\varphi+\rho^2\sin^2\varphi}\; d\varphi\, d\rho$ $\qquad dx = d\rho$

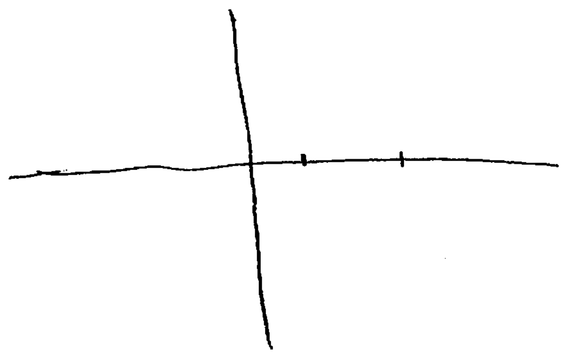
$\iint_D \rho\, d\varphi\, d\rho = \int_0^3 \rho\, d\rho \int_0^{\pi/2} d\varphi =$

$= \int_0^3 \rho\, d\rho \left(\varphi\Big|_0^{\pi/2} \right) = \frac{\pi}{2}\int_0^3 \rho\, d\varphi = \frac{\pi}{2}\,\frac{\rho^2}{2}\Big|_0^3 =$

$= \frac{\pi}{4}(9-0) = \frac{9\pi}{4}$

(2.) $\iint_D \ln(x^2+y^2)\,dx\,dy$, $D = \{(x,y) \mid e^2 \le x^2+y^2 \le e^4\}$

?



6.5 a).

1.) $x = 4y - y^2$, $x + y = 6 \longrightarrow y = 6 - x$

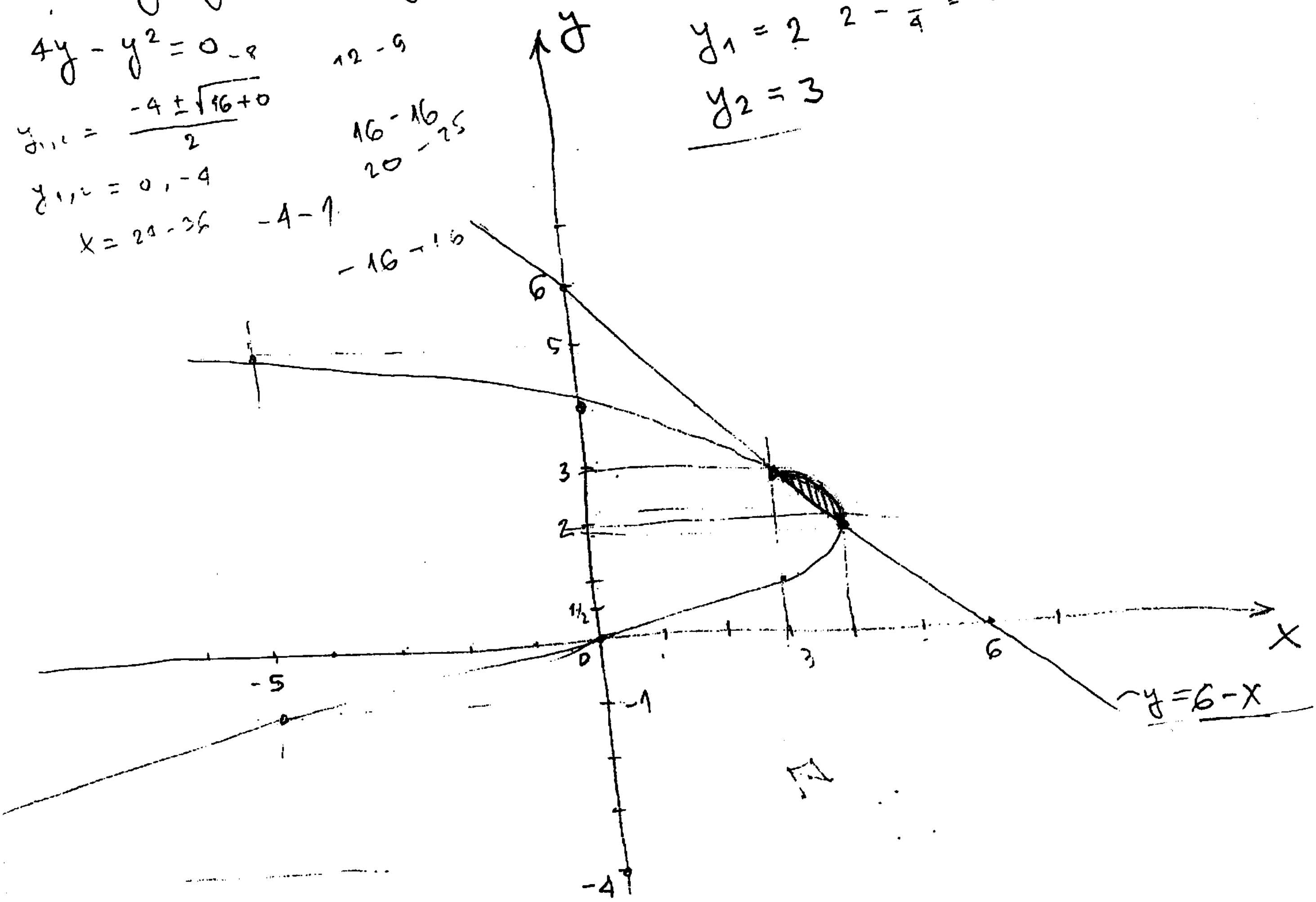
$4y - y^2 = 0$

$y_{1,2} = \frac{-4 \pm \sqrt{16+0}}{2}$

$y_{1,2} = 0, -4$

$y_1 = 2$

$y_2 = 3$



$x = 4(6-x) - (6-x)^2 = 24 - 4x - (36 - 12x + x^2) =$

$= 24 - 4x - 36 + 12x - x^2 = -12 + 8x - x^2$

$x - 8x + x^2 + 12 = 0 \qquad x^2 - 7x + 12 = 0$

$x_{1,2} = \frac{7 \pm \sqrt{49-48}}{2} = \frac{7 \pm 1}{2}$

$\boxed{x_1 = 4 \quad x_2 = 3}$

$$P = \int_2^3 dy \int_{6-y}^{4y-y^2} dx = \int_2^3 dy\,(4y - y^2 - 6 + y) =$$

$$= \int_2^3 (5y - y^2 - 6)\,dy = 5\frac{y^2}{2}\Big|_2^3 - \frac{y^3}{3}\Big|_2^3 - 6y\Big|_2^3 = \frac{25}{2} - \frac{19}{3} - 6 = \frac{1}{6}$$

$= \frac{5}{2}(9-4) - \frac{1}{3}(27-8) - 6(3-2)$

$9-6$

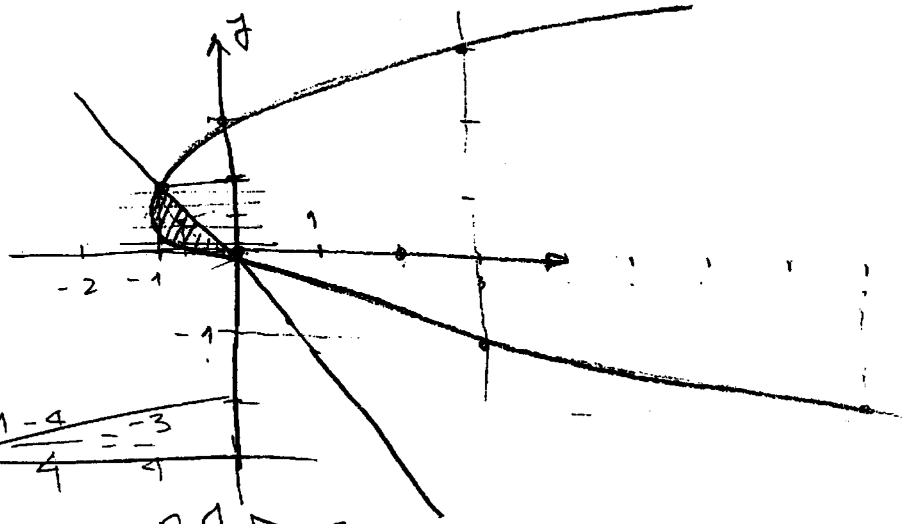
$1 - 7$

(2.) $x = y^2 - 2y$, $x + y = 0$ $\qquad x = -y$

$-y = y^2 - 2y$

$0 = y^2 - y$

$y_{1,2} = \frac{1 \pm \sqrt{1}}{2} = \begin{cases} 1 \\ 0 \end{cases}$



$$P = \int_0^1 dy \int_{y^2-2y}^{-y} dx = \cancel{\int_0^1 dy (\ldots)} =$$

$$= \int_0^1 dy \, (-y - y^2 + 2y) = \int_0^1 (y - y^2)\, dy =$$

$$= \frac{y^2}{2}\Big|_0^1 - \frac{y^3}{3}\Big|_0^1 = \frac{1}{2}(1-0) - \frac{1}{3}(1-0) =$$

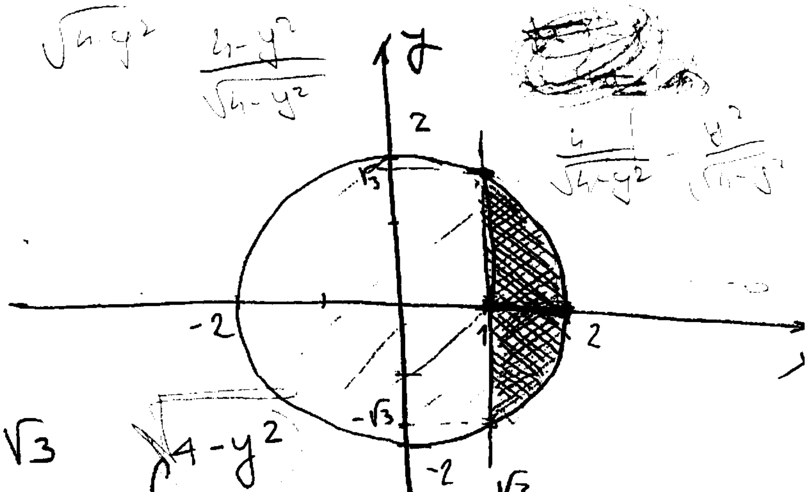
$$\frac{3-2}{6} = \frac{1}{6}$$ ✓

(3.) $x^2 + y^2 \le 4$, $x \ge 1$

$1 + y^2 \le 4 \qquad x^2 = 4 - y^2$

$y^2 = 3$

$\boxed{y = \pm\sqrt{3}}$



$$P = 2 \cdot \int_0^{\sqrt{3}} dy \int_1^{\sqrt{4-y^2}} dx = 2 \int_0^{\sqrt{3}} dy \, (4 - y^2 - 1)$$

$$= 2 \cdot \int_0^{\sqrt{3}} (3 - y^2)\, dy = 6\sqrt{3} - \frac{2}{3} \cancel{3} = \underline{\underline{6\sqrt{3} - 2}}$$

14

4. $y = 2 - x$, $y^2 = 4x + 4$ $\longrightarrow$ $4x = y^2 - 4$, $x = \frac{y^2-4}{4}$

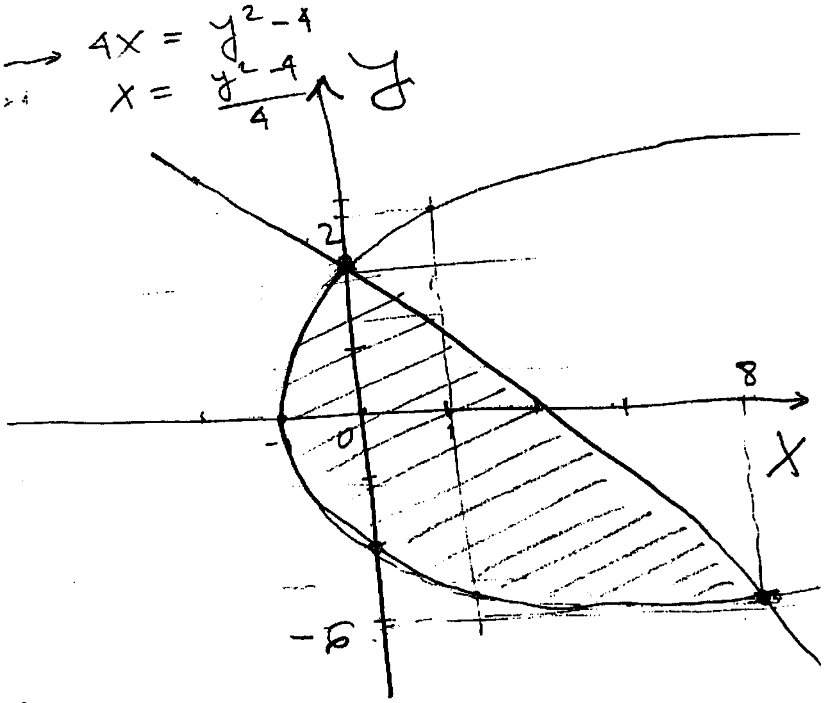
$2 - y = x$

$4 - 4x + x^2 = 4x + 4$ $\pm 2\sqrt{2}$

$x^2 - 8x = 0$

$x(x-8) = 0$

$$\boxed{\begin{array}{l} x_1 = 0 \ \wedge \ x_2 = 8 \\ y_1 = 2 \ \wedge \ y_2 = -6 \end{array}}$$



$$P = \int_{-6}^{2} dy \int_{\frac{y^2-4}{4}}^{2-y} dx = \int_{-6}^{2} dy \left(2 - y - \frac{y^2-4}{4}\right) =$$

$$= \int_{-6}^{2} \frac{\cancel{4} - 4y - y^2 \cancel{- 4}}{4} dy = \int_{-6}^{2} \frac{-4y - y^2}{4} dy =$$

$$= -\frac{1}{4} \int_{-6}^{2} (4y + y^2)\, dy = -\frac{1}{\cancel{4}} \cdot \cancel{4} \cdot \frac{y^2}{2} \Big|_{-6}^{2} - \frac{1}{4} \frac{y^3}{3} \Big|_{-6}^{2} =$$

$$= -\frac{1}{2}(4 - 36) - \frac{1}{12}(8 + 216) =$$

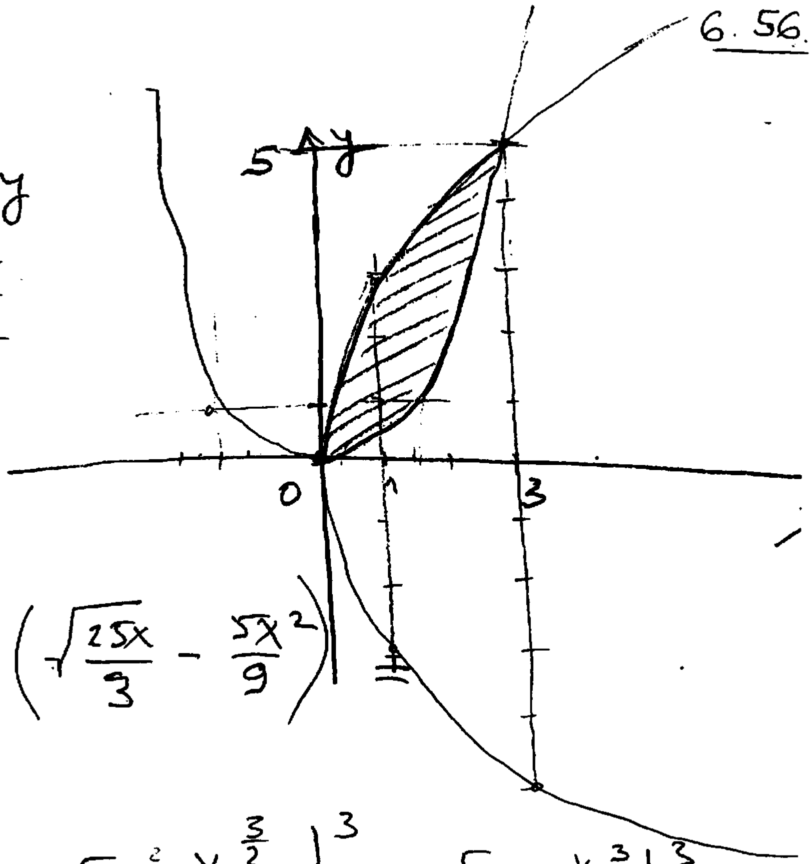
$$= 16 - \frac{224}{12} = 16 - \frac{56}{3} = \frac{48 - 56}{3} = \boxed{-\frac{8}{3}}$$

(5.) $3y^2=25x, \quad 5x^2=9y$

$y^2=\frac{25x}{3} \qquad x^2=\frac{9y}{5}$

$\frac{5}{\sqrt3}$

$y=\sqrt{\frac{25x}{3}} \qquad y=\frac{5x^2}{9}$



$$P=\int_0^3 dx\int_{\frac{5x^2}{9}}^{\sqrt{\frac{25x}{3}}} dy=\int_0^3 dx\left(\sqrt{\frac{25x}{3}}-\frac{5x^2}{9}\right)$$

$x^{\frac12+1}=\frac32$

$$=\int_0^3\left(\frac{5}{\sqrt3}\sqrt{x}-\frac{5}{9}x^2\right)dx=\frac{5}{\sqrt3}\,\frac{x^{\frac32}}{\frac32}\Bigg|_0^3-\frac{5}{9}\,\frac{x^3}{3}\Bigg|_0^3=$$

$$=\cancel{\frac{10}{3\sqrt3}}\left(\cancel{\sqrt{27}}-0\right)-\frac{5}{27}(27-0)=10-5=\underline{\underline{5}}$$

$y^2=4x-x^2$

(6.) $\underline{x^2+y^2\le 4x, \quad 2x\le y^2} \to y^2=2x$

$x^2-4x+y^2\leq 0 \qquad 2\le y\ddot{o}\le y^2-2x$

$2\sqrt2$

$(x-2)^2+y^2\le 4 \qquad \boxed{y\pm\sqrt2} \qquad 1\ \ -6$

4

$x^2+2x=4x \qquad y^2=4-1$

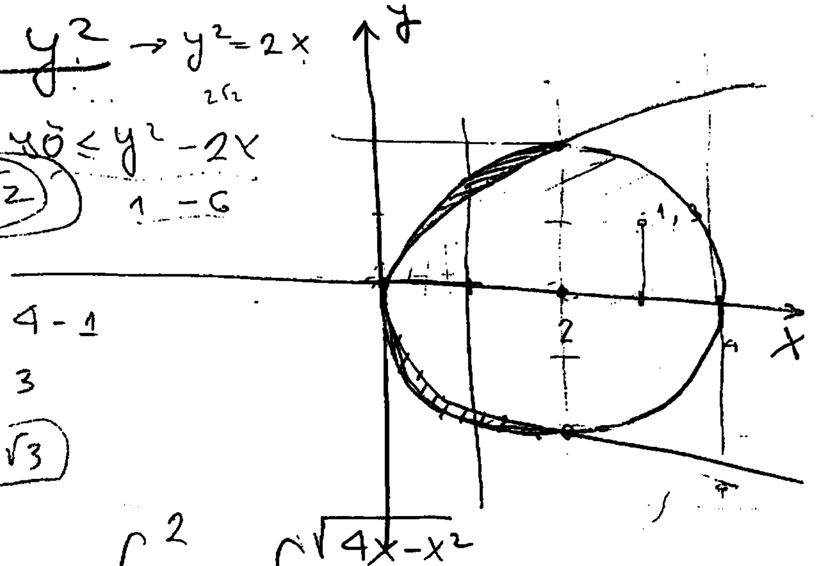
$y=3$

$x^2-2x=0 \qquad \boxed{y=\pm\sqrt3}$

$x(x-2)=0$

$x_1=0 \quad \cancel{x}_2=2$

$y_1=0 \quad y=\pm2$



$$P=2\cdot\int_0^2 dx\int_{\sqrt{2x}}^{\sqrt{4x-x^2}} dy=$$

$$=2\cdot\int_0^2 dx\left(\sqrt{4x-x^2}-\sqrt{2x}\right)=$$

6.3/.

$$= 2\cdot\int_0^2\left(\sqrt{4x-x^2}-\sqrt{2x}\right)dx = 2\cdot\left(\int_0^2\sqrt{4x-x^2}\,dx-\int_0^2\sqrt{2x}\,dx\right)=$$

$=$

$$\int\sqrt{4x-x^2}\,dx = \int\frac{4x-x^2}{\sqrt{4x-x^2}}\,dx =$$

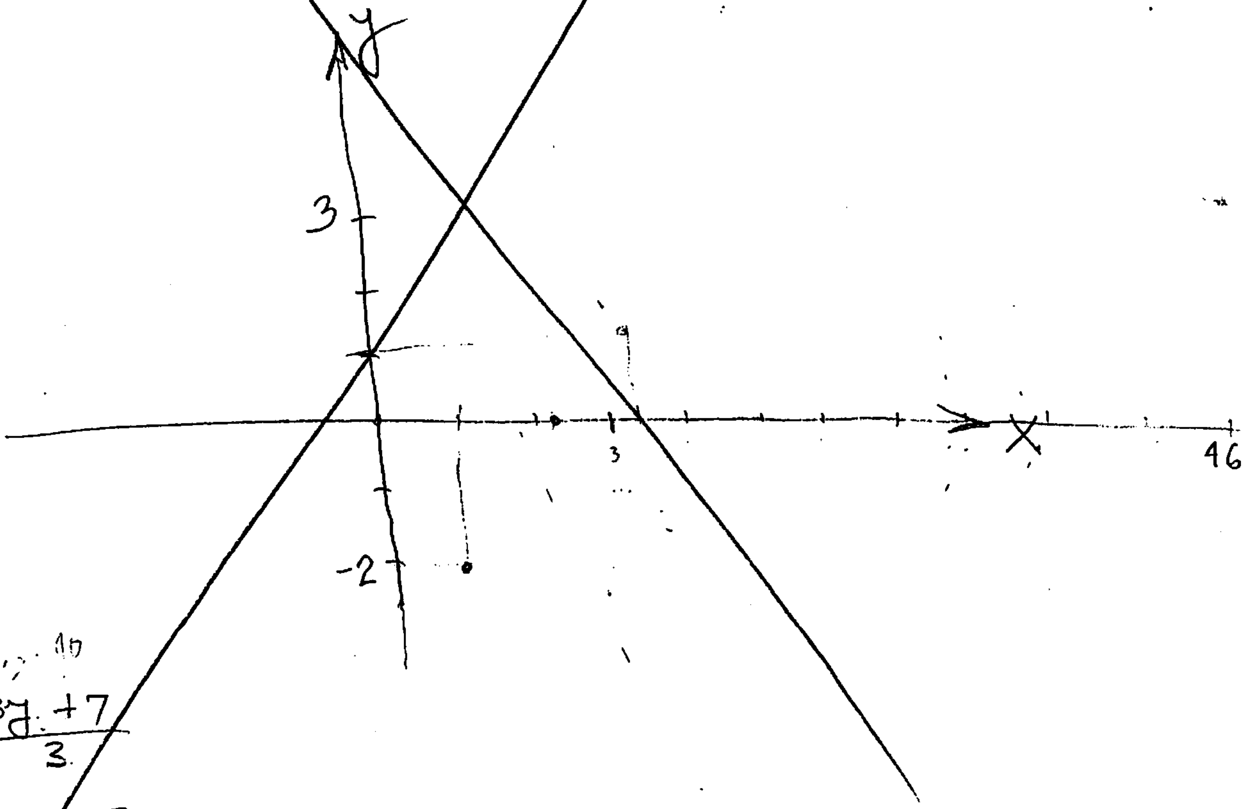
7) $y^2+2y-3x+1=0$, $3x-3y-7=0$

$3x = 3y+7 \rightarrow x_1 = 16$, $x_2 = 1$

$y^2+2y+1 = 3x$

$y^2+2y+1 = 3y+7$

$y^2-y-6=0$ $\qquad y_{1,2} = \frac{1\pm\sqrt{1+24}}{2} = \frac{1\pm5}{2}$ $\begin{cases}3\\-2\end{cases}$



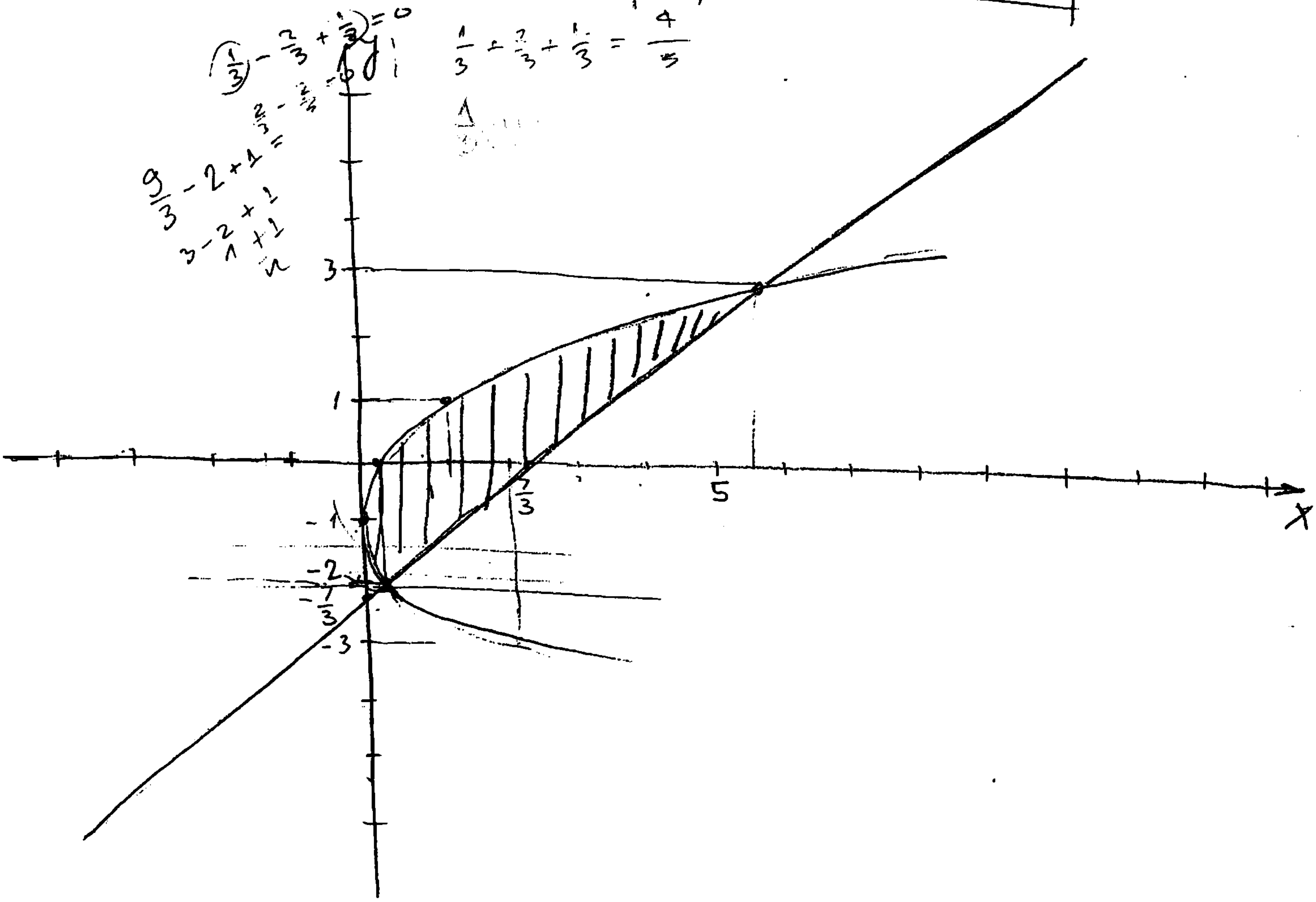
$x = \frac{3y+7}{3}$

$x = y+\frac{7}{3}$

7. $y^2+2y-3x+1=0$, $3x-3y-7=0$

$3x = y^2+2y+1$; $3x = 3y+7$

$\boxed{x=\frac{1}{3}y^2+\frac{2}{3}y+\frac{1}{3}}$; $\boxed{x = y+\frac{7}{3}}$



$\frac{1}{3}y^2+\frac{2}{3}y+\frac{1}{3} = y+\frac{7}{3}$

$\frac{1}{3}y^2+\frac{2}{3}y-y = \frac{7}{3}-\frac{1}{3}$

$\frac{1}{3}y^2-\frac{1}{3}y = 2$

$\frac{1}{3}(y^2-y) = 2$

$y^2-y = 6$

$y^2-y-6=0$

$y_{1,2} = \frac{1\pm\sqrt{1+24}}{2} = \frac{1\pm 5}{2}$ — 3, -2

$x_1 = 3+\frac{7}{3} = \frac{9+7}{3} = \frac{16}{3}$

$x_2 = -2+\frac{7}{3} = \frac{-6+7}{3} = \frac{1}{3}$

6.59.

1.

6.5.

$$P=\int_{-2}^{3}dy\int_{\frac{1}{3}y^2+\frac{2}{3}y+\frac{1}{3}}^{y+\frac{2}{3}}dx=\int_{-2}^{3}\left(y+\frac{2}{3}-\frac{1}{3}y^2-\frac{2}{3}y-\frac{1}{3}\right)dy=$$

180
−38
142
+15
157

9−4

27−8

$$=\int_{-2}^{3}\left(\frac{1}{3}y-\frac{1}{3}y^2+2\right)dy=\frac{1}{3}\,\frac{y^2}{2}\Big|_{-2}^{3}-\frac{1}{3}\,\frac{y^3}{3}\Big|_{-2}^{3}+2y\Big|_{-2}^{3}=$$

$$=\frac{5}{6}-\frac{19}{9}+10=\frac{15-38+180}{18}=\boxed{\frac{157}{18}}$$ ✓

2−5

18−15

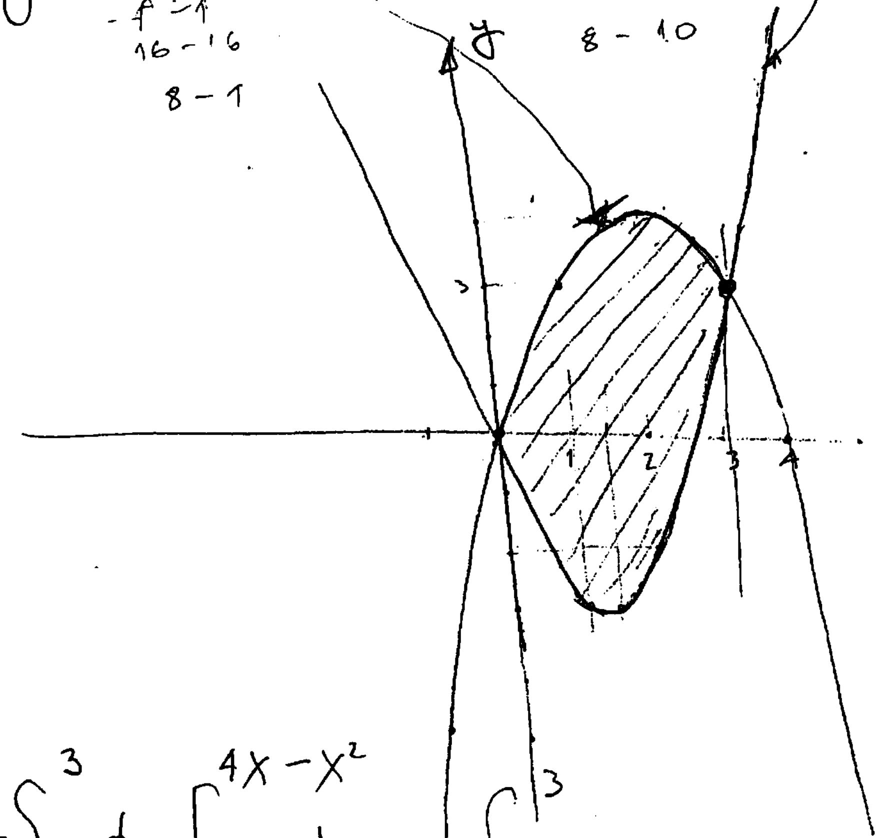
12−9 = 3

8.) $y=4x-x^2$, $y=2x^2-5x$

−4 −1
16−16
8−1

32 − 20
8 − 10

18−15



$4x-x^2=2x^2-5x$

$4x+5x-x^2-2x^2=0$

$9x-3x^2=0 \quad /:3$

$3x-x^2=0$

$x_{1,2}=\frac{-3\pm\sqrt{9}}{-2}$

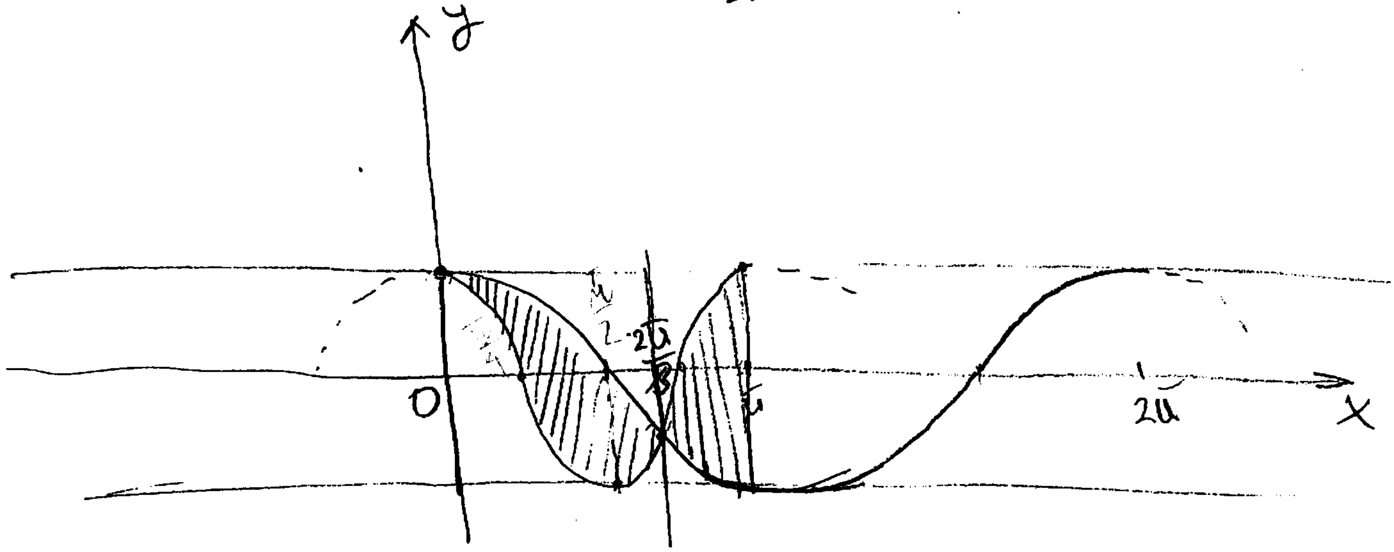
$\boxed{x_1=0 \quad x_2=3}$

$\boxed{y_1=0 \quad y_2=3}$

$$P=\int_{0}^{3}dx\int_{2x^2-5x}^{4x-x^2}dy=\int_{0}^{3}\left(4x-x^2-2x^2+5x\right)dx=$$

$$=\int_{0}^{3}\left(9x-3x^2\right)dx=9\cdot\frac{x^2}{2}\Big|_{0}^{3}-3\,\frac{x^3}{3}\Big|_{0}^{3}=\frac{81}{2}-27$$

9\. $y \le \cos x,\ y \ge \cos 2x \quad (0 \le x \le \pi)$



$\cos x = \cos 2x$

$\cos x = \cos^2 x - \sin^2 x$

$\cos x = 2\cos^2 x - 1$

$2\cos^2 x - \cos x - 1 = 0$

$t = \cos x$

$\arccos 1 = x_1 = 0$

$\arccos\left(-\frac{1}{2}\right) = x_2 = \frac{2\pi}{3}$

$2t^2 - t - 1 = 0$

$t_{1/2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \quad t_1 = 1,\ t_2 = -\frac{1}{2}$

$$P = \int_0^{\frac{2\pi}{3}} dx \int_{\cos 2x}^{\cos x} dy + \int_{\frac{2\pi}{3}}^{\pi} dx \int_{\cos x}^{\cos 2x} dy =$$

$$= \int_0^{\frac{2\pi}{3}} (\cos x - \cos 2x)\,dx + \int_{\frac{2\pi}{3}}^{\pi} (\cos 2x - \cos x)\,dx =$$

$$= \sin x \Big|_0^{\frac{2\pi}{3}} - \frac{1}{2}\sin 2x \Big|_0^{\frac{2\pi}{3}} + \frac{1}{2}\sin 2x \Big|_{\frac{2\pi}{3}}^{\pi} - \sin x \Big|_{\frac{2\pi}{3}}^{\pi} =$$

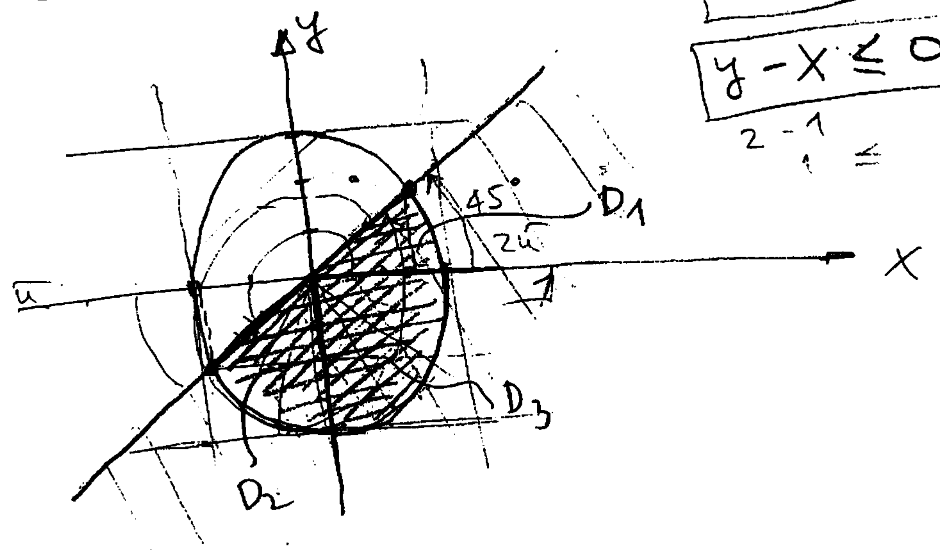
14. $9x^2+4y^2 \le 36 \quad y \le x$

$$\boxed{\frac{x^2}{4}+\frac{y^2}{9} \le 1}$$

$$\boxed{y - x \le 0}$$

$2 \quad -1$

$1 \quad \le$



$\pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

$9x^2 + 4x^2 = 36$

$13x^2 = 36 \quad x^2 = \frac{36}{13} \Rightarrow \boxed{x = \pm\frac{6}{\sqrt{13}}}$

$\boxed{y = x = \pm\frac{6}{\sqrt{13}}} \Rightarrow$

$$\boxed{\begin{aligned} x &= \rho\cos\varphi \\ y &= \rho\sin\varphi \end{aligned}}$$

$9\rho^2\cos^2\varphi + 4\rho^2\sin^2\varphi = 36$

$\rho^2(9\cos^2\varphi + 4\sin^2\varphi) = 36$

$$\boxed{\rho^2 = \frac{36}{9\cos^2\varphi + 4\sin^2\varphi}}$$

$D_1: \begin{cases} 0 \le \varphi \le \pi/4 \\ 0 \le \rho \le \frac{6}{\sqrt{9\cos^2\varphi + 4\sin^2\varphi}} \end{cases}$

$D_2: \begin{cases} \frac{5\pi}{4} \le \varphi \le \frac{3\pi}{2} \\ 0 \le \rho \le \frac{6}{-\sqrt{9\cos^2\varphi + 4\sin^2\varphi}} \end{cases}$

D_2

$D_3: \begin{cases} \frac{3\pi}{2} \le \varphi \le 2\pi \\ 0 \le \rho \le \frac{6}{-\sqrt{9\cos^2\varphi + 4\sin^2\varphi}} \end{cases}$

$J = \begin{vmatrix} x'_\rho & x'_\varphi \\ y'_\rho & y'_\varphi \end{vmatrix} = \rho$

$$P = \int_0^{\frac{6}{\sqrt{9\cos^2\varphi+4\sin^2\varphi}}} \rho\, d\rho \int_0^{\frac{\pi}{4}} d\varphi + \int_0^{-\frac{6}{\sqrt{\cancel{9\cos^2\varphi+4\sin^2\varphi}}}} \rho\, d\rho \int^{2\pi} d\varphi = \frac{2\pi - \frac{5\pi}{4}}{\frac{8\pi - 5\pi}{4}}$$

$$= \int_0 \rho\, d\rho \left(\frac{\pi}{4}\right) + \int_0 \rho\, d\rho \left(2\pi - \frac{5\pi}{4}\right)\frac{3\pi}{4} =$$

$$P = \frac{\overline{u}}{4}\int_0^{\frac{6}{\sqrt{9\cos^2\varphi+4\sin\varphi}}} \rho\, d\rho + \frac{3\overline{u}}{4}\int_0^{\frac{6}{\sqrt{9\cos^2\varphi+4\sin\varphi}}} \rho\, d\rho =$$

$$= \frac{\overline{u}}{4}\,\frac{\rho^2}{2}\Big|_0^{\frac{6}{\sqrt{9\cos^2\varphi\ldots}}} + \frac{3u}{8}\,\rho^2\Big|_0^{\frac{6}{\sqrt{96\ldots}}} =$$

$36:2=18$

$\frac{2}{16}$

$$= \frac{\overline{u}}{8}\cdot\frac{36}{9\cos^2\varphi+4\sin^2\varphi} + \frac{3\overline{u}}{8}\cdot\frac{36}{9\cos^2\varphi+4\sin^2\varphi} =$$

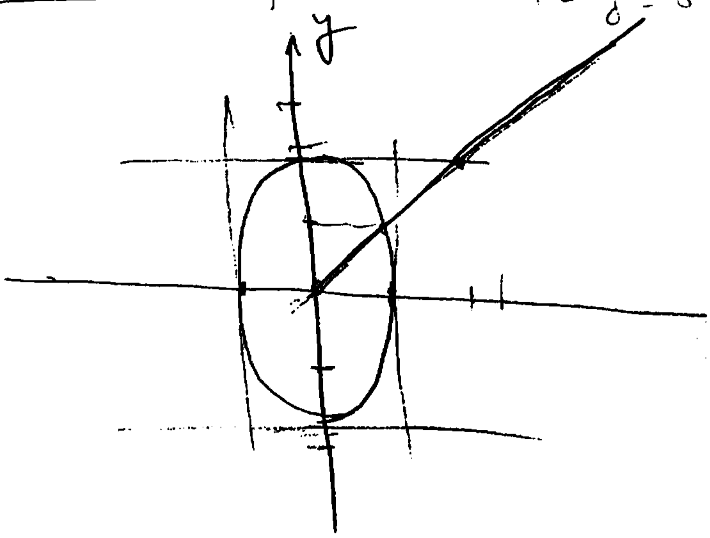
$$P = \frac{\cancel{36}^{\,18}}{9\cos^2\varphi+4\sin^2\varphi}\cdot\frac{\cancel{4}\overline{u}}{8\cancel{2}} = \boxed{\frac{18}{9\cos^2\varphi+4\sin^2\varphi}}$$ ✓

(18.) $3x^2+y^2\le 3 \,/:3$ $|x|\le|y|$ $\rightarrow$ $|x|-|y|\le 0$

$x^2+\frac{y^2}{3}\le 1 \rightarrow \boxed{a=1 \quad b=\sqrt{3}}$

$\sqrt{x}-\sqrt{y}=0$

$1-y=0$

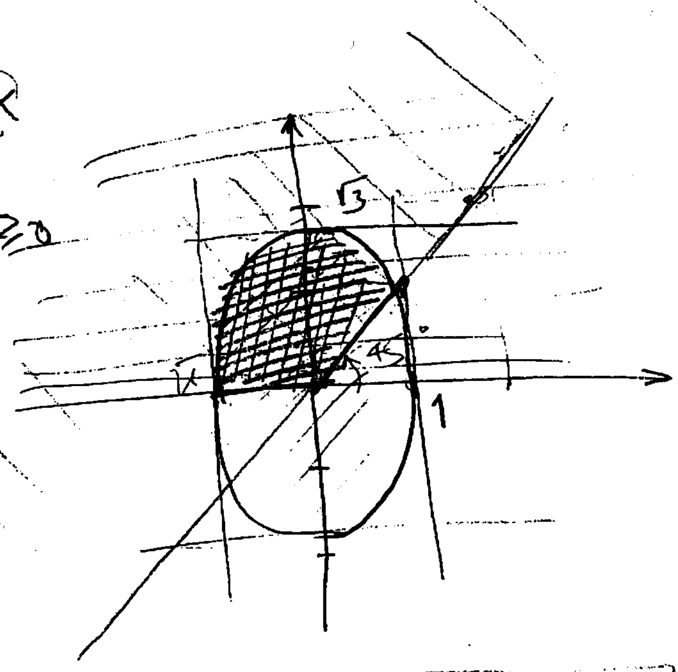


?

0

17. $3x^2+y^2 \le 3$, $0 \le y \ge x$

$x^2 + \frac{y^2}{3} \le 1$ $\qquad$ $y - x \ge 0$



$x = \rho\cos\varphi$ $\qquad$ $\boxed{J = \rho}$

$y = \rho\sin\varphi$

$3\rho^2\cos^2\varphi + \rho^2\sin^2\varphi = 3$

$\rho^2(3\cos^2\varphi + \sin^2\varphi) = 3$

$\rho^2 = \frac{3}{3\cos^2\varphi + \sin^2\varphi} \rightarrow \boxed{\rho = \sqrt{\frac{3}{3\cos^2\varphi + \sin^2\varphi}}}$

$$D = \left\{ \begin{array}{l} 0 \le \rho \le \sqrt{\frac{3}{3\cos^2\varphi + \sin^2\varphi}} \\ \frac{\pi}{4} \le \varphi \le \pi \end{array} \right\}$$

$$P = \int_{\frac{\pi}{4}}^{\pi} d\varphi \int_0^{\sqrt{\frac{3}{3\cos^2\varphi+\sin^2\varphi}}} \rho\, d\rho = \int_{\frac{\pi}{4}}^{\pi} d\varphi \left[\frac{\frac{3}{3\cos^2\varphi+\sin^2\varphi}}{2}\right] =$$

$$= \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{d\varphi}{3\cos^2\varphi + \sin^2\varphi} =$$

$3\cos^2\varphi + 1 - \cos^2\varphi$

$\boxed{1 + 2\cos^2\varphi}$ $\rightarrow \cos\varphi = t$

$\cos^2\varphi + \sin^2\varphi$

$$\cancel{= \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{1\, d\varphi}{1 + 2\cos^2\varphi} = \frac{3}{4}\int_{\frac{\pi}{4}}^{\pi} \frac{d\varphi}{\frac{1}{2} + \cos^2\varphi} =}$$

$$P = \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{d\varphi}{3\cos^2\varphi + \sin^2\varphi} = \left\{ \begin{array}{l} \operatorname{tg}\varphi = t \\ d\varphi = \frac{dt}{1+t^2} \\ \cos\varphi = \frac{1}{\sqrt{1+t^2}} \end{array} \quad \sin\varphi = \frac{t}{\sqrt{1+t^2}} \right\}$$

$$= P = \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{\frac{dt}{1+t^2}}{3\cdot\frac{1}{1+t^2} + \frac{t^2}{1+t^2}} = \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{\frac{dt}{\cancel{1+t^2}}}{\frac{3+t^2}{\cancel{1+t^2}}} =$$

$$= \frac{3}{2}\int_{\frac{\pi}{4}}^{\pi} \frac{dt}{3+t^2} = \frac{3}{2}\cdot\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{t}{\sqrt{3}}\Bigg|_{\frac{\pi}{4}}^{\pi} =$$

$$= \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{\sqrt{3}}}\operatorname{arctg}\frac{\operatorname{tg}\varphi}{\sqrt{3}}\Bigg|_{\frac{\pi}{4}}^{\pi} \quad \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2}\operatorname{arctg}\frac{\operatorname{tg}\varphi}{\sqrt{3}}\Bigg|_{\frac{\pi}{4}}^{\pi} = \frac{\sqrt{3}}{2}\left(\cancel{\operatorname{arctg}\frac{0}{\sqrt{3}}} - \operatorname{arctg}\frac{1}{\sqrt{3}}\right)$$

$$\boxed{P = -\frac{\sqrt{3}}{2}\operatorname{arctg}\frac{1}{\sqrt{3}}}$$

$$\boxed{P = \frac{\sqrt{3}}{2}\operatorname{arctg}\frac{1}{\sqrt{3}}}$$

~~66.6)~~ 65. b)

(1.) $\Gamma: x + 2y + 3z = 1$ [у првом октанту]

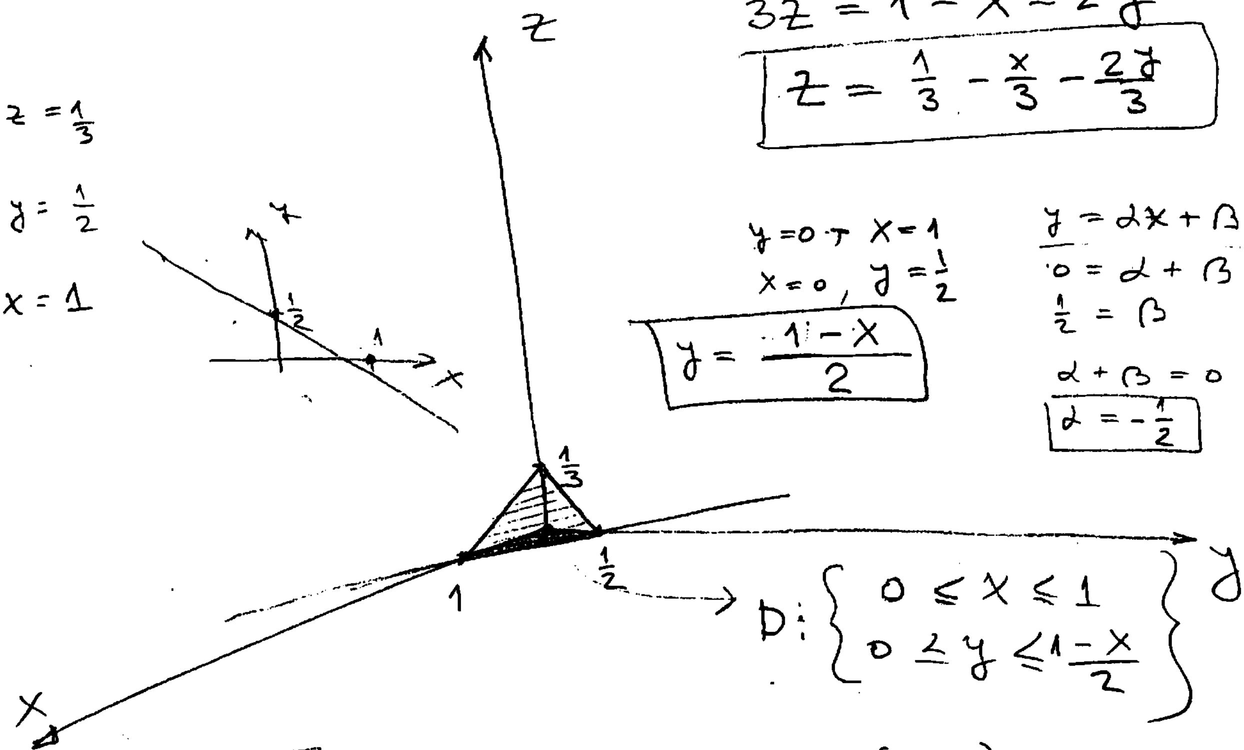
$$3z = 1 - x - 2y$$

$$\boxed{z = \frac{1}{3} - \frac{x}{3} - \frac{2y}{3}}$$

$\left.\begin{matrix} x=0 \\ y=0 \end{matrix}\right\} z = \frac{1}{3}$

$\left.\begin{matrix} x=0 \\ z=0 \end{matrix}\right\} y = \frac{1}{2}$

$\left.\begin{matrix} y=0 \\ z=0 \end{matrix}\right\} x = 1$



$y = 0 \to x = 1$

$x = 0, \ y = \frac{1}{2}$

$$\boxed{y = \frac{1-x}{2}}$$

$y = \alpha x + \beta$

$0 = \alpha + \beta$

$\frac{1}{2} = \beta$

$\alpha + \beta = 0$

$\boxed{\alpha = -\frac{1}{2}}$

$$D: \left\{ \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le \frac{1-x}{2} \end{matrix} \right\}$$

$S: \ z = z(x,y)$

$p = z_x$

$q = z_y$

$T_1: \ z_1 = z_1(x,y)$

$T_2: \ z_2 = z_2(x,y)$

$$V(T) = \iint_D (z_2 - z_1)\, dx\, dy$$

$$P(S) = \iint_D \sqrt{1 + p^2 + q^2}\, dx\, dy$$

$1 + \frac{1}{9} + \frac{4}{9} = \frac{9+1+4}{9} = \frac{14}{9}$

$$\boxed{\begin{matrix} p = -\frac{1}{3} \\ q = -\frac{2}{3} \end{matrix}} \longrightarrow P(S) = \int_0^1 dx \int_0^{\frac{1-x}{2}} \sqrt{\frac{14}{9}}\, dy =$$

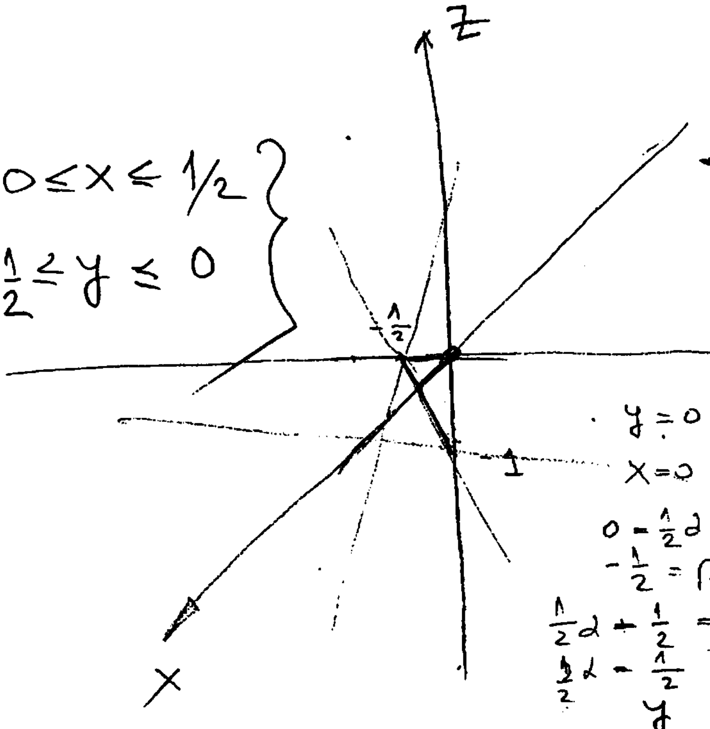
$$= \int_0^1 dx \left(\sqrt{\frac{14}{9}} \cdot \left(\frac{1-x}{2} - 0 \right) \right) = \sqrt{\frac{14}{9}} \int_0^1 \frac{1-x}{2}\, dx =$$

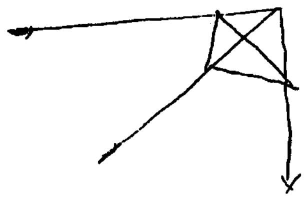
$$= \sqrt{\frac{14}{9}} \cdot \frac{1}{2} x \Big|_0^1 - \sqrt{\frac{14}{9}} \cdot \frac{1}{2} \cdot \frac{y^2}{2} \Big|_0^1 = \frac{\sqrt{14}}{6} - \frac{\sqrt{14}}{24} = \frac{4\sqrt{14} - \sqrt{14}}{24} =$$

$$= \frac{3\sqrt{14}}{24} = \frac{\sqrt{14}}{8}$$

② $\Gamma: \ 2x-2y-z=1 \quad [x=0,\ y=0,\ z=0]$

$$D=\left\{\begin{array}{l} 0\le x\le 1/2 \\ x-\frac{1}{2}\le y\le 0\end{array}\right\}$$



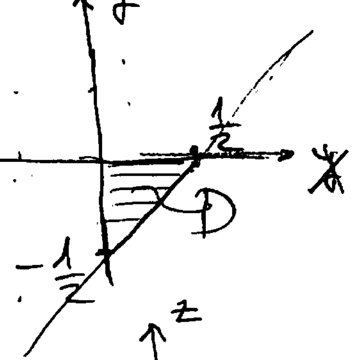


$y=0 \quad x=\frac{1}{2}$
$x=0 \quad y=-\frac{1}{2}$

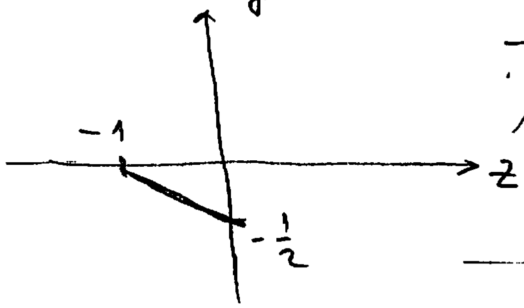
$y=\alpha x+\beta$
$\boxed{y=x-\frac{1}{2}}$

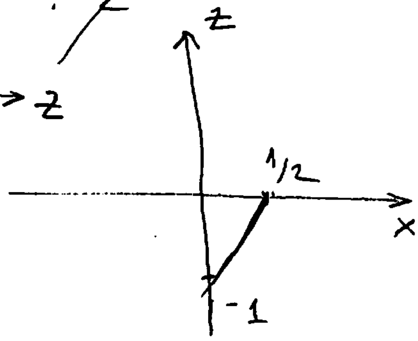
$0=\frac{1}{2}\alpha+\beta$
$-\frac{1}{2}=\beta$

$\frac{1}{2}\alpha+\frac{1}{2}=0$
$\frac{1}{2}\alpha-\frac{1}{2}$ $\boxed{\alpha=1}$



$$\boxed{2x-2y-1=z}$$





$p=2$
$q=-2$

$$P(S)=\int_0^{1/2}dx\int_{x-\frac{1}{2}}^{0}\sqrt{1+4+4}\,dy=3\int_0^{1/2}dx\left(0-x+\frac{1}{2}\right)=$$

$$=3\int_0^{1/2}\left(\frac{1}{2}-x\right)dx=3\cdot\frac{1}{2}x\Big|_0^{1/2}-3\cdot\frac{x^2}{2}\Big|_0^{1/2}=$$

$$=\frac{3}{2}\cdot\frac{1}{2}-\frac{3}{2}\cdot\frac{1}{4}=\frac{3}{4}-\frac{3}{8}=\frac{6-3}{8}=\frac{3}{8}$$

$-x-2y-1=z$

3.) $\Gamma: -x-2y-z=1;\ [x=0,\ y=0,\ z=0]$

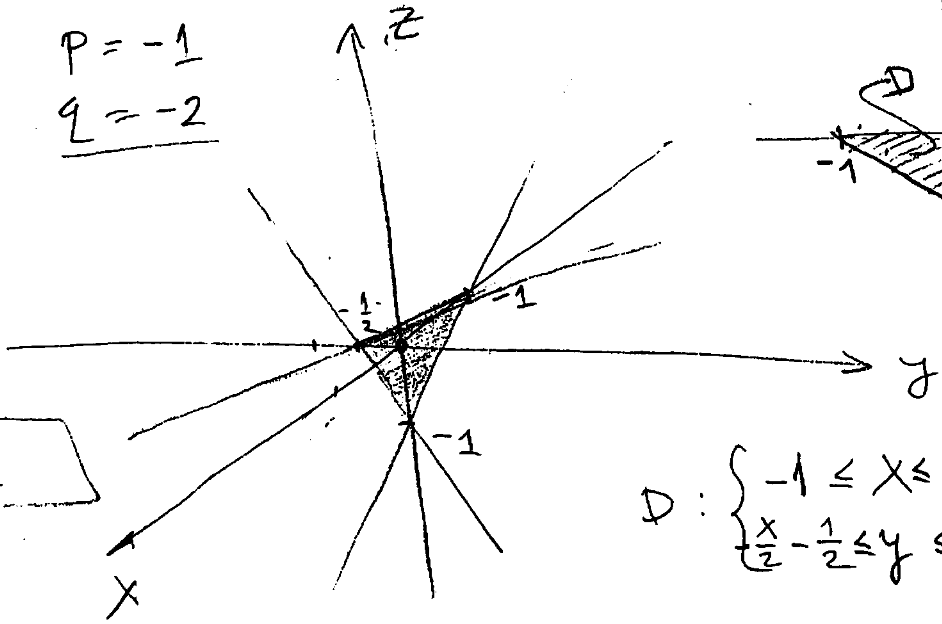
$y=0\ x=-1$
$x=0\ y=-\frac{1}{2}$

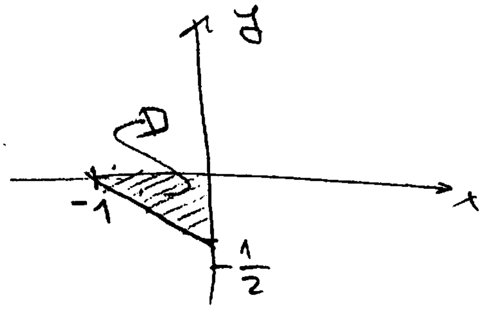
$p=-1$
$\underline{q=-2}$

$0=-\alpha+\beta$
$-\frac{1}{2}=\beta$

$0=-\alpha+\frac{1}{2}$
$\alpha=-\frac{1}{2}$

$\boxed{y=-\frac{x}{2}-\frac{1}{2}}$





$D: \begin{cases} -1 \le x \le 0 \\ \frac{x}{2}-\frac{1}{2} \le y \le 0 \end{cases}$

$$P(S)=\int_{-1}^{0} dx \int_{-\frac{x}{2}-\frac{1}{2}}^{0} \sqrt{1+1+4}\, dy = \sqrt{6}\int_{-1}^{0} dx \left(0+\frac{x}{2}+\frac{1}{2}\right)=$$

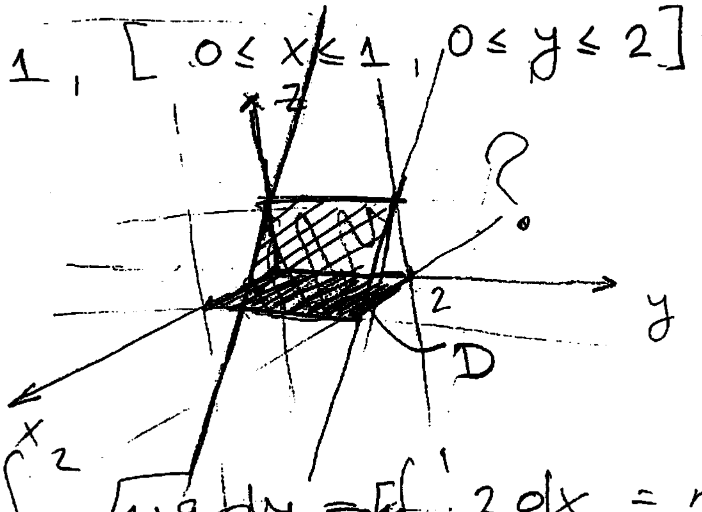
$$=\sqrt{6}\cdot\frac{1}{2}\cdot\frac{x^2}{2}\Big|_{-1}^{0}+\sqrt{6}\cdot\frac{1}{2}x\Big|_{-1}^{0}=\frac{\sqrt{6}}{4}(0-1)+\frac{\sqrt{6}}{2}(0+1)$$

$$=-\frac{\sqrt{6}}{4}+\frac{\sqrt{6}}{2}=\frac{-\sqrt{6}+2\sqrt{6}}{4}=\underline{\frac{\sqrt{6}}{4}}\ \| \checkmark$$

4.) $\Gamma: 3x+z=1,\ [0\le x\le 1,\ 0\le y\le 2]$

$\underline{z=1-3x}$

$\boxed{\begin{matrix} p=-3 \\ q=0 \end{matrix}}$



$$P(S)=\int_0^1 dx\int_0^2 \sqrt{1+9}\, dy=\sqrt{10}\int_0^1 2\,dx = 2\sqrt{10}\,x\Big|_0^1=\underline{\underline{2\sqrt{10}}}$$

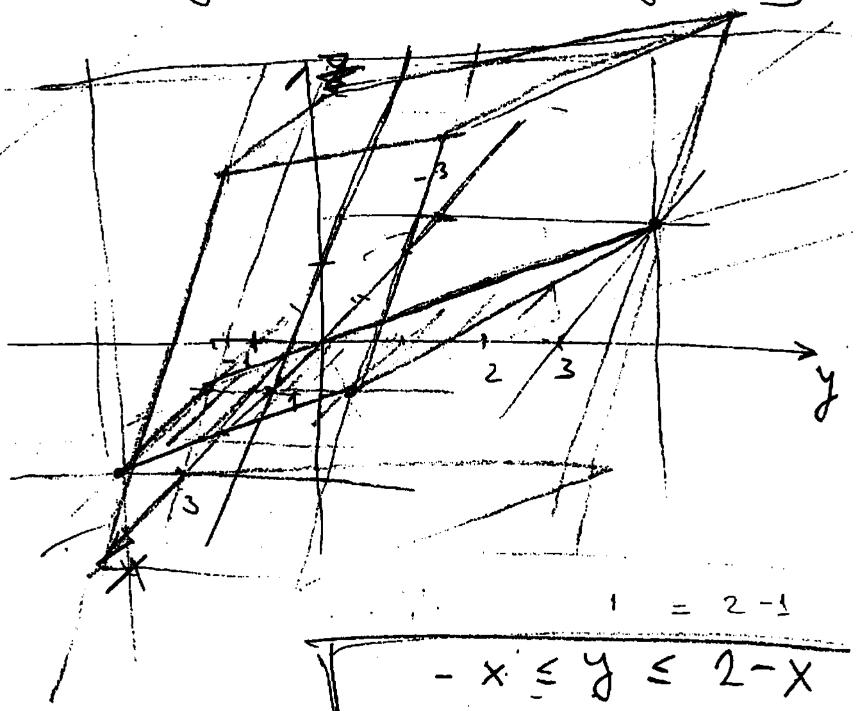
⑤ $\Gamma: 3x + z = 1 \quad [0 \le x+y \le 2 \quad 1 \le x+2y \le 3]$

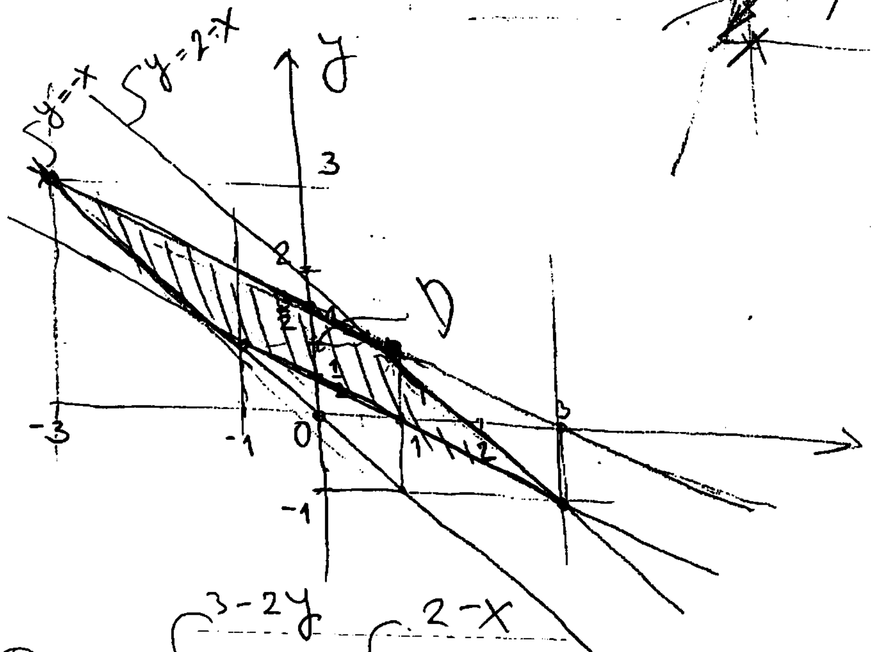
$z = 1 - 3x$

$y = -x \quad y = 2-x$

$x = 1-2y$

$x = 3-2y$





$1 = 2-1$

$$\boxed{\begin{aligned} -x &\le y \le 2-x \\ 1-2y &\le x \le 3-2y \end{aligned}}$$

$x' = 3 - 2 \cdot 3 = -3$

$p = 3$

$q = 0$

$$P(S) = \int_{1-2y}^{3-2y} dx \int_{-x}^{2-x} \sqrt{1+9}\, dy = \sqrt{10} \int_{1-2y}^{3-2y} (2-x+x)\, dx$$

$$P(S) = 2\sqrt{10}\,(3 - \cancel{2y} - 1 + \cancel{2y}) = 4\sqrt{10}$$

✓

6.) $\Gamma: 3x + z = 1 \quad [3 \le y \le 5,\ 2y \le z \le 3y]$

$\boxed{x = \frac{1}{3} - \frac{z}{3}}$

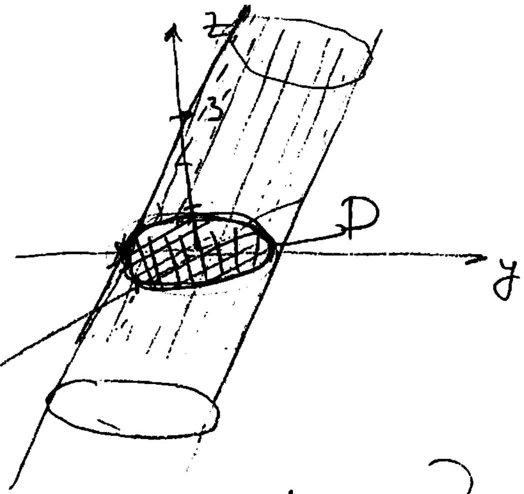
$\boxed{\begin{array}{l} p = 0 \\ q = -\frac{1}{3} \end{array}}$

$$P(S) = \int_3^5 dy \int_{2y}^{3y} \sqrt{1 + \frac{1}{9}}\, dz = \frac{\sqrt{10}}{3} \int_3^5 (\underbrace{3y - 2y}_{y})\, dy =$$

$\frac{9+1}{9}$

$$= \frac{\sqrt{10}}{3} \cdot \frac{y^2}{2} \Big|_3^5 = \frac{\sqrt{10}}{3} \cdot \left(\frac{25}{2} - \frac{9}{2} \right) = \frac{16}{2} \cdot \frac{\sqrt{10}}{3}$$

$$\boxed{P(S) = \frac{16\sqrt{10}}{6}}$$



7.) $\Gamma: 3y - z = -3 \quad [x^2 + y^2 = 1]$

$\boxed{z = 3y + 3}$

$p = 0$

$q = 3$

$$D = \left\{ \begin{array}{c} -1 \le x \le 1 \\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2} \end{array} \right\}$$

$$P(S) = \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1+9}\, dy = \sqrt{10} \int_{-1}^{1} dx \cdot \left(\sqrt{1-x^2} + \sqrt{1-x^2} \right) =$$

$$= \sqrt{10} \int_{-1}^{1} 2\sqrt{1-x^2}\, dx = 2 \cdot \sqrt{10} \int_{-1}^{1} \sqrt{1-x^2}\, dx$$

6.70.

$$\int \sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}\,dx = (Ax+B)\sqrt{1-x^2} + \lambda \int \frac{dx}{\sqrt{1-x^2}}$$

$$\frac{1-x^2}{\sqrt{1-x^2}} = A\sqrt{1-x^2} + (Ax+B)\frac{-\cancel{2}x}{\cancel{2}\sqrt{1-x^2}} + \frac{\lambda}{\sqrt{1-x^2}}$$

$$1-x^2 = A(1-x^2) - (Ax+B)x + \lambda$$

$$1-x^2 = A - Ax^2 - Ax^2 - Bx + \lambda$$

$$-2A = -1 \rightarrow \boxed{A = \frac{1}{2}}$$

$$\boxed{B = 0}$$

$$A + \lambda = 1 \rightarrow \boxed{\lambda = 1 - \frac{1}{2} = \frac{1}{2}}$$

$$\boxed{\int \sqrt{1-x^2}\,dx = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x}$$

$$P(S) = 2\sqrt{10}\cdot\left(\frac{1}{2}x\sqrt{1-x^2}\Big|_{-1}^{1} + \frac{1}{2}\arcsin x\Big|_{-1}^{1}\right) =$$

$$= 2\cdot\sqrt{10}\left(\frac{1}{2}(0) + \frac{1}{2}(\arcsin 1 + \arcsin(-1))\right)$$

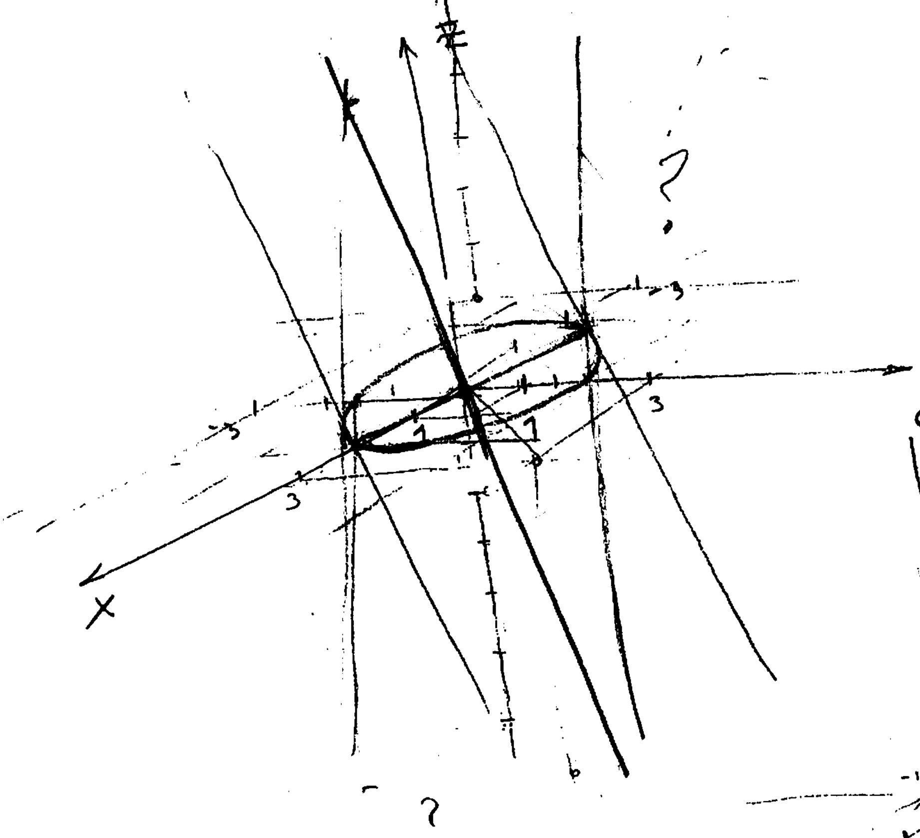
$$\frac{\pi}{2} + \frac{\pi}{2} = \cancel{2}\frac{\pi}{\cancel{2}}$$

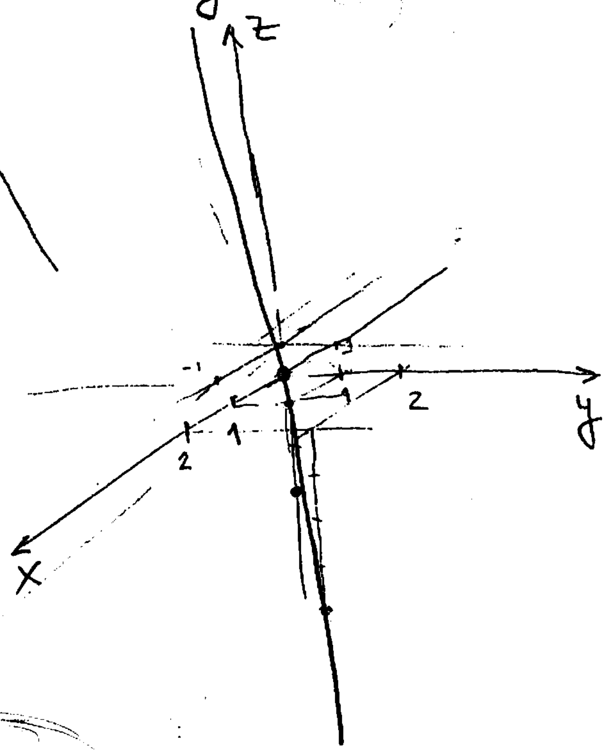
$$\boxed{P(S) = 2\sqrt{10}\cdot\frac{\pi}{2} = \pi\sqrt{10}}$$

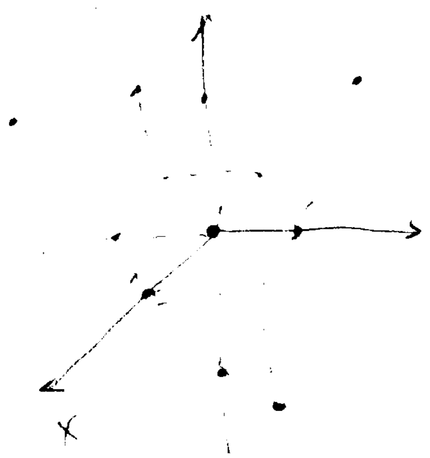
8) $r: x+y+z=0 \quad [x^2+2y^2 \leq 4]$

$z=-x-y$

$\boxed{\frac{x^2}{4}+\frac{y^2}{2}\leq 1}$ Elipse





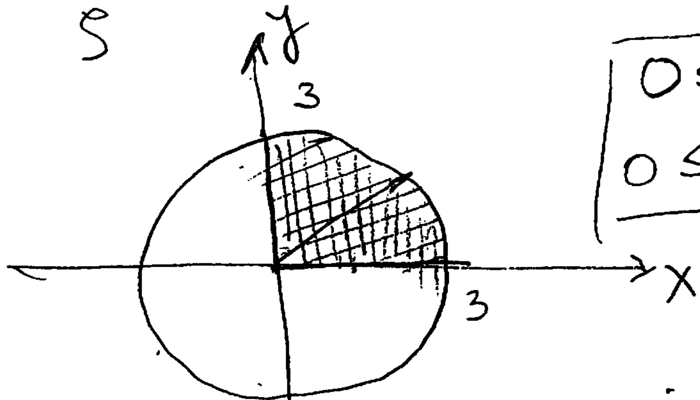


наставак:

в) Прелазећи на поларне координате израчунати двојни интеграл:

1. $\iint\limits_D \sqrt{x^2+y^2}\,dx\,dy$, D је четвртина круга $x^2+y^2=9$ у првом квадранту.

$\sqrt{\rho^2\cos^2\varphi+\rho^2\sin^2\varphi} = \rho$



$$\boxed{\begin{array}{l} 0\le\rho\le 3 \\ 0\le\varphi\le\frac{\pi}{2}\end{array}}$$

$$x=\rho\cos\varphi$$
$$y=\rho\sin\varphi$$

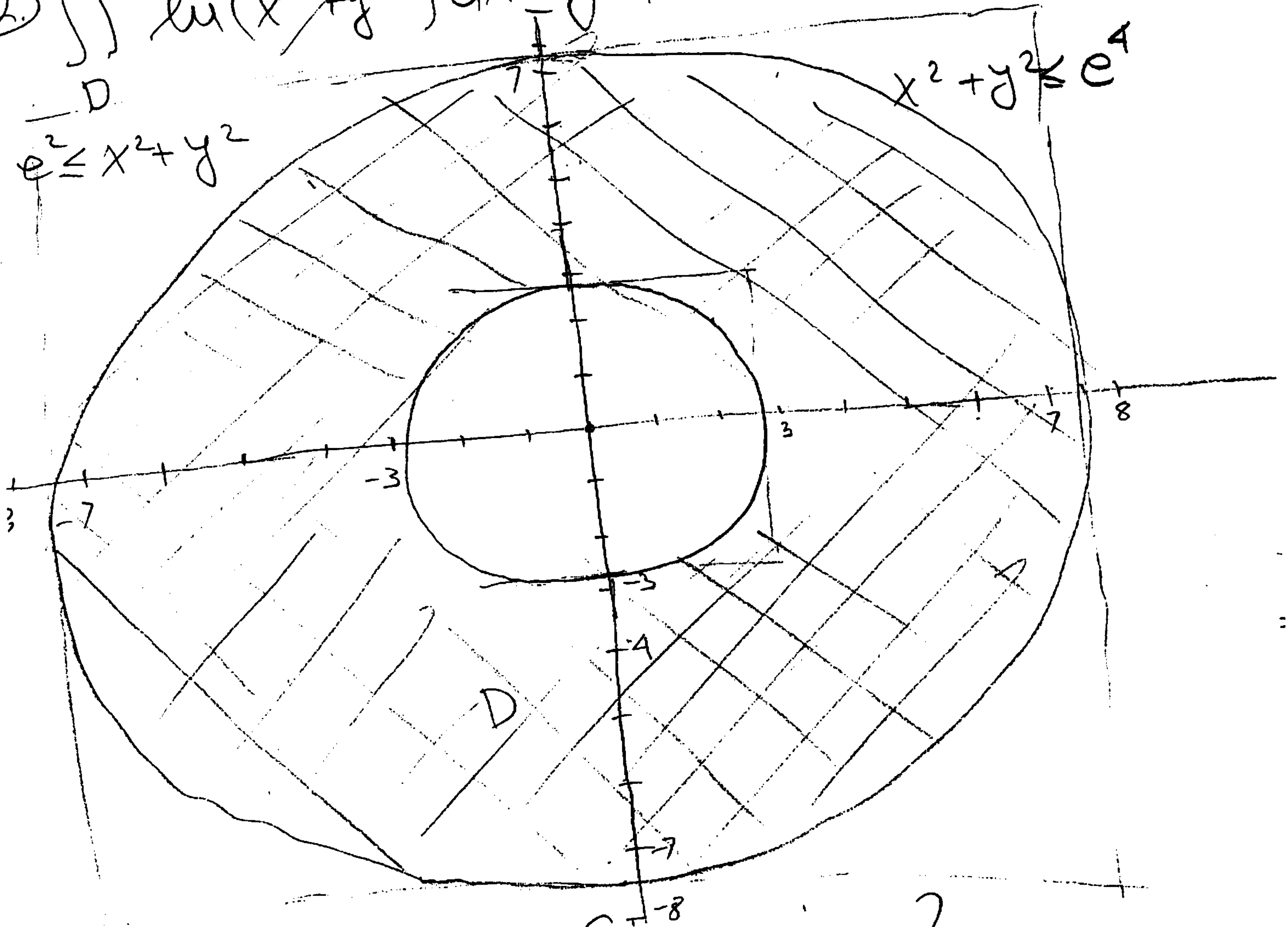
$$J=\begin{vmatrix} x'_\rho & x'_\varphi \\ y'_\rho & y'_\varphi \end{vmatrix} = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{vmatrix} = \rho\cos^2\varphi+\rho\sin^2\varphi = \underline{\underline{\rho}}$$

$$\int_0^{\frac{\pi}{2}} d\varphi \int_0^3 \rho^2\,d\rho = \int_0^{\frac{\pi}{2}} \left(\frac{\rho^3}{3}\Big|_0^3\right) d\varphi =$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{3}(27-0)\,d\varphi = 9\int_0^{\frac{\pi}{2}} d\varphi = 9\cdot\varphi\Big|_0^{\frac{\pi}{2}} =$$

$$= \underline{\underline{\frac{9\pi}{2}}}$$

2.) $\iint\limits_D \ln(x^2+y^2)\,dx\,dy$, $D=\{(x,y)\mid e^2\le x^2+y^2\le e^4\}$



$x=\rho\cos\varphi$
$y=\rho\sin\varphi$
$|J|=\rho$

D' : $\left\{\begin{array}{l} e\le\rho\le e^2 \\ 0\le\varphi\le 2\pi \end{array}\right\}$

$$\iint\limits_{D'} \ln\rho^2\cdot\rho\,d\varphi\,d\rho=\int_e^{e^2}d\rho\int_0^{2\pi}\rho\ln\rho^2\,d\varphi=$$

$$=\int_e^{e^2}d\rho\left(\rho\ln\rho^2\cdot\varphi\Big|_0^{2\pi}\right)=2\pi\int_e^{e^2}\rho\ln\rho^2\,d\rho$$

$$\int \rho \ln \rho^2 \, d\rho = \begin{cases} \ln \rho^2 = u \to \frac{1}{\rho^2} \cdot 2\rho \, d\rho = du \\ v = \frac{\rho^2}{2} \end{cases}$$ 6.14

$$= \frac{\rho^2}{2} \ln \rho^2 - \int \frac{1}{\cancel{\rho^2}} \cancel{2}\rho \cdot \frac{\cancel{\rho^2}}{\cancel{2}} \, d\rho = \quad \boxed{e \cong 2{,}73}$$

$$= \frac{\rho^2}{2} \ln \rho^2 - \frac{\rho^2}{2} = \underline{\frac{\rho^2}{2} (\ln \rho^2 - 1) + c}$$

$$= \int_e^{e^2} \rho \ln \rho^2 \, d\rho = \frac{\rho^2}{2} \ln \rho^2 \Big|_e^{e^2} - \frac{\rho^2}{2} \Big|_e^{e^2} =$$

$$= \frac{1}{2} (e^4 \ln e^4 - e^2 \ln e^2) - \frac{1}{2} (e^4 - e^2) =$$

$$= \frac{1}{2} (e^4 \cdot 4 - e^2 \cdot 2) - \frac{1}{2} (e^4 - e^2) =$$

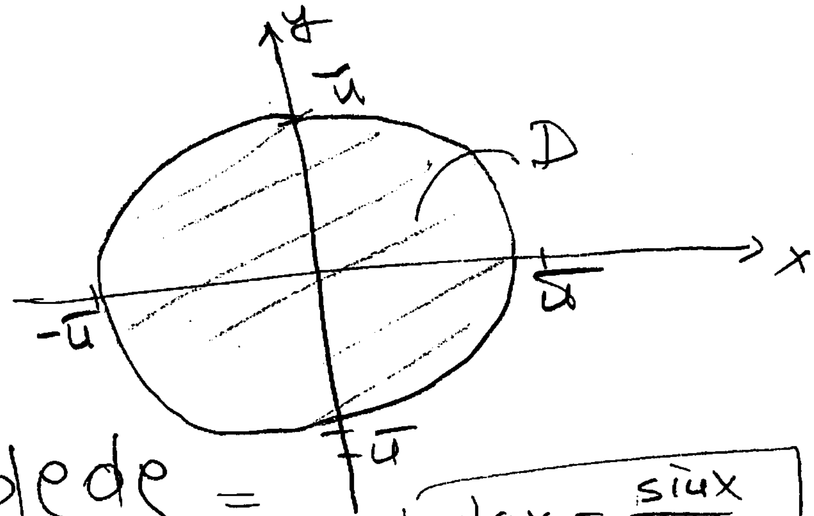
2,73

$$= 2e^4 - e^2 - \frac{e^4}{2} + \frac{e^2}{2} = \underline{\frac{3}{2} e^4 - \frac{1}{2} e^2} \Big| \cdot \ln$$

$\frac{4}{2}$

3. $\iint_D \left(1-\frac{y^2}{x^2}\right)dx\,dy$, $D=\{(x,y)\mid x^2+y^2\le \pi^2\}$

$$D' = \begin{Bmatrix} 0\le\rho\le\pi \\ 0\le\varphi\le 2\pi\end{Bmatrix}$$



$$\iint_{D'}\left(1-\frac{\cancel{\rho^2}\sin^2\varphi}{\cancel{\rho^2}\cos^2\varphi}\right)\cdot\rho\,d\varphi\,d\rho =$$

$$\boxed{tg\,x=\frac{\sin x}{\cos x}}$$

$$=\int_0^\pi d\rho\int_0^{2\pi}(1-tg\,\varphi)\rho\,d\varphi=$$

$$=\int_0^\pi d\rho\int_0^{2\pi}(\rho-\rho\,tg\,\varphi)\,d\varphi=$$

$$=\int_0^\pi d\rho\left[\rho\varphi\Big|_0^{2\pi}-\cancel{\rho\ln|\cos\varphi|\Big|_0^{2\pi}}\right]=$$

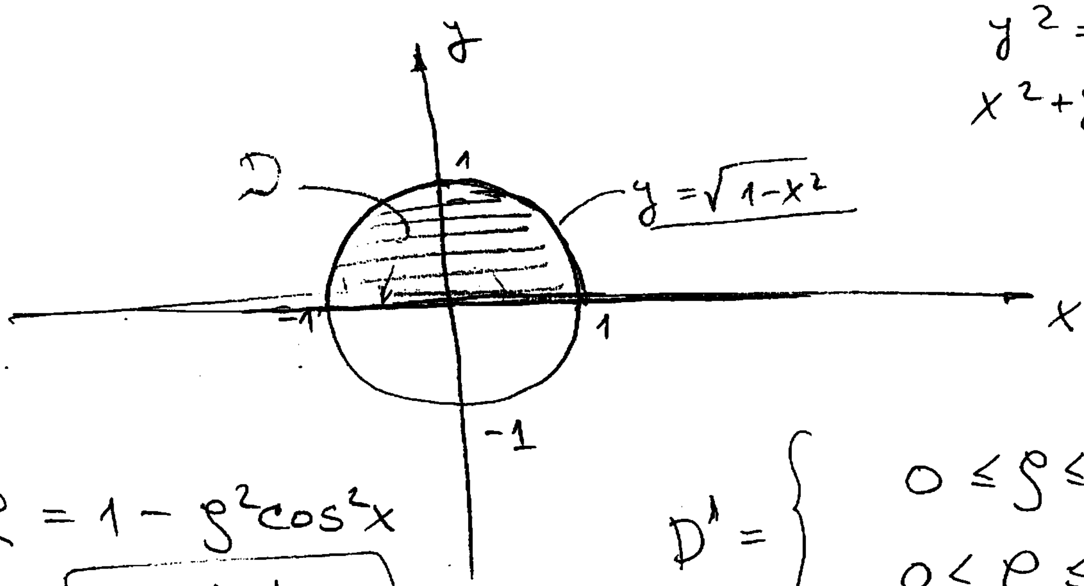
$$=\int_0^\pi d\rho\,[2\pi\rho-0]=2\pi\int_0^\pi\rho\,d\rho=2\pi\frac{\rho^2}{2}\Big|_0^\pi=$$

$$=\boxed{\pi^3}\ ✓$$

4\. $\iint\limits_D \frac{dxdy}{1+x^2+y^2}$, D је ограничена линијама $y=0$ и $y=\sqrt{1-x^2}$

$y^2=1-x^2$

$x^2+y^2=1$



$\rho^2\sin^2\varphi = 1-\rho^2\cos^2 x$

$\rho^2=1 \quad \boxed{\rho=|1|=1}$

$D' = \left\{ \begin{array}{l} 0 \le \rho \le 1 \\ 0 \le \varphi \le \pi \end{array} \right\}$

t^{-1+1}

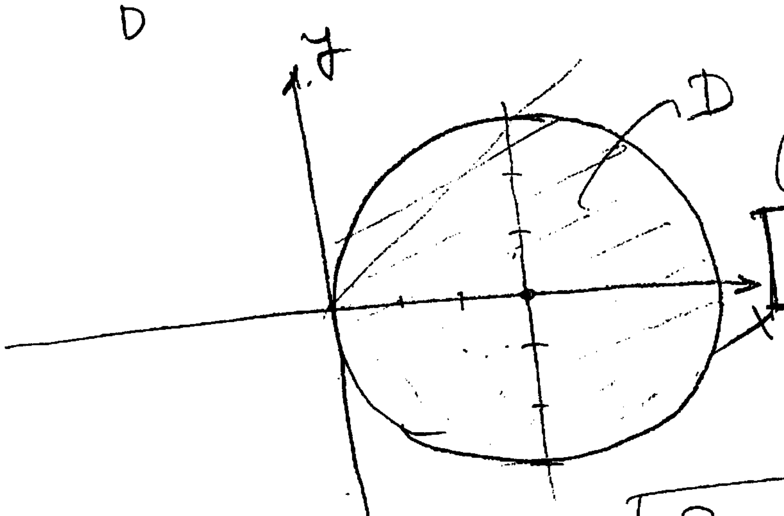
$$\iint\limits_{D'} \frac{\rho d\varphi d\rho}{1+(\rho^2\cos^2\varphi+\rho^2\sin^2\varphi} = \iint\limits_{D'} \frac{\rho d\varphi d\rho}{1+\rho^2} =$$

$$= \int_0^\pi d\varphi \int_0^1 \frac{\rho}{1+\rho^2}\, d\rho = \left\{ \begin{array}{l} 1+\rho^2=t \\ 2\rho d\rho = dt \\ \rho d\rho = \frac{1}{2}dt \end{array} \right\} =$$

$$= \int_0^\pi d\varphi \int_0^1 \frac{\frac{1}{2}dt}{t} = \frac{1}{2}\int_0^\pi d\varphi \left[\ln(1+\rho^2)\Big|_0^1 \right] =$$

$$= \frac{1}{2}\int_0^\pi (\ln 2 - \cancel{\ln 1})\, d\varphi = \frac{\ln 2}{2}\varphi\Big|_0^\pi = \frac{\ln 2}{2}\pi$$

5. $\iint\limits_D (x^2+y^2)\,dx\,dy$, $D=\{(x,y)\mid x^2+y^2\le 6x\}$



$x^2+y^2-6x\le 0$

$(x-3)^2+y^2-9\le 0$

$\boxed{(x-3)^2+y^2\le 9}$

$\rho^{\cancel{2}}=6\cancel{\rho}\cos\varphi \rightarrow \boxed{\rho=6\cos\varphi}$

$$D':\begin{cases}0\le\rho\le 6\cos\varphi\\ -\frac{\pi}{2}\le\varphi\le\frac{\pi}{2}\end{cases}$$

$$\iint\limits_{D'}\rho^2\cdot\rho\,d\rho\,d\varphi=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^{6\cos\varphi}\rho^3\,d\rho=$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\left[\frac{\rho^4}{4}\Big|_0^{6\cos\varphi}\right]=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}324\cos^4\varphi\,d\varphi=$$

$$=324\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^4\varphi\,d\varphi=324\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos^2\varphi)^2\,d\varphi=324\,\cdot$$

$$=324\cdot\left(\frac{\cos^3\varphi\sin\varphi}{4}+\frac{3}{4}\int\cos^2\varphi\,d\varphi\right)=$$

$$=324\left(\frac{\cos^3\varphi\sin\varphi}{4}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}+\frac{3}{4}\left(\frac{1}{4}\sin 2\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}+\frac{1}{2}\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right)\right)=$$

$$= \cancel{81 \cdot 0} + \frac{243}{4}\left(\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right) + \frac{243}{2}\left(\underbrace{\frac{\pi}{2}+\frac{\pi}{2}}_{2\frac{\pi}{2}}\right) =$$

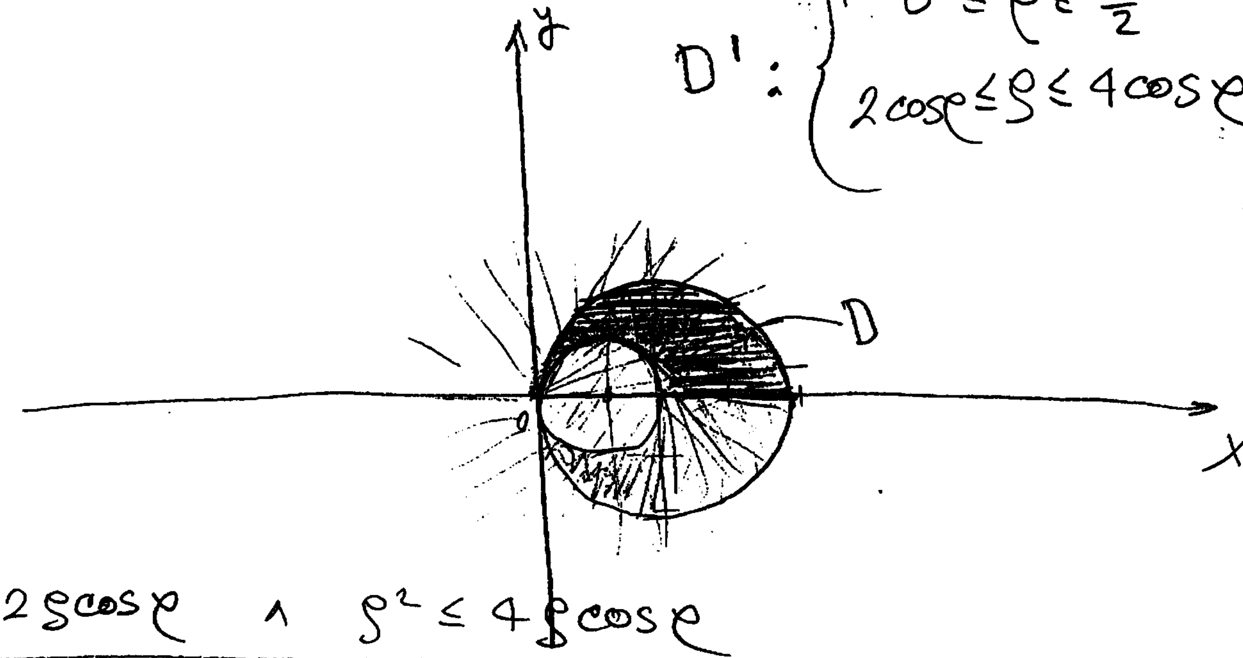
$$= \frac{243}{2} + \frac{243\pi}{2} = \underline{\frac{243}{2}(1+\pi)}$$

6. $\iint_D (x^2+y^2)\,dx\,dy$, $D = \{(x,y) \mid x^2+y^2 \geq 2x \land x^2+y^2 \leq 4x \land y \geq 0\}$

$(x-1)^2 + y^2 - 1 \geq 0 \quad \land \quad (x-2)^2 + y^2 - 4 \leq 0 \quad \land \quad y \geq 0$

$\boxed{(x-1)^2 + y^2 \geq 1} \quad \land \quad \boxed{(x-2)^2 + y^2 \leq 4}$

$$D' : \begin{cases} 0 \leq \varphi \leq \frac{\pi}{2} \\ 2\cos\varphi \leq \rho \leq 4\cos\varphi \end{cases}$$



$\rho^2 \geq 2\rho\cos\varphi \quad \land \quad \rho^2 \leq 4\rho\cos\varphi$

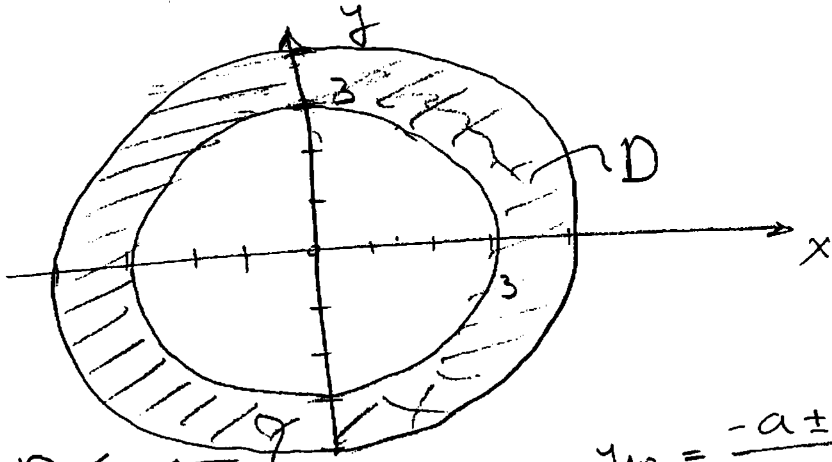
$\boxed{\rho \geq 2\cos\varphi \quad \land \quad \rho \leq 4\cos\varphi}$

$$\iint_{D'} \rho^2 \cdot \rho \, d\rho \, d\varphi = \int_0^{\frac{\pi}{2}} d\varphi \int_{2\cos\varphi}^{4\cos\varphi} \rho^3 \, d\rho =$$

$$= \int_0^{\frac{\pi}{2}} d\varphi \left(\frac{\rho^4}{4} \Big|_{2\cos\varphi}^{4\cos\varphi} \right) = \int_0^{\pi/2} (64\cos^4\varphi - 4\cos^4\varphi)\,d\varphi$$

$$= 60\int_0^{\frac{\pi}{2}} \cos^4\varphi \, d\varphi = \dots$$

7) $\iint_D (x^2+y^2)\,dx\,dy$; $D = \{(x,y) \mid 9 \le x^2+y^2 \le 16\}$



$$D' : \begin{cases} 0 \le \varphi \le 2\pi \\ 3 \le \rho \le 4 \end{cases}$$

$ay - x^2 \le 0$

$y - x^2 \le 0$

$3 - 4 \le$

$y_{1,2} = \dfrac{-a \pm \sqrt{a^2+8a^2}}{2}$ $2a$

$y_{1,2} = \dfrac{-a \pm 3a}{2}$ $-\frac{4a}{2}$

$\boxed{y_1 = a \mid \cancel{y_2 = -2a}}$

$ay + y^2 = 2a^2$

$y^2 + ay - 2a^2 = 0$

$\rho^2 = 2a$

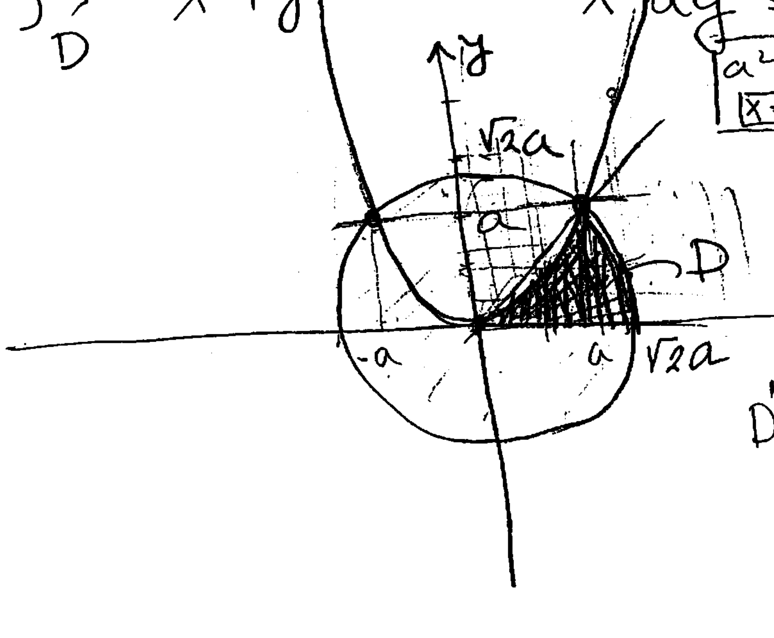
$$\int_3^4 d\rho \int_0^{2\pi} \rho^2 \, d\varphi =$$

8) $\iint_D \dfrac{x\,dx\,dy}{x^2+y^2}$, $D = \{(x,y) \mid x^2+y^2 \le 2a^2 \wedge$
$\wedge\ ay \le x^2 \wedge y \ge 0 \wedge x \ge 0\}$

$\boxed{\begin{matrix} a^2 = x^2 \\ \boxed{x = \pm a} \end{matrix}}$

$a\cancel{\rho}\sin\varphi = \rho^{\cancel{2}}\cos^2\varphi$

$\boxed{\dfrac{a\sin\varphi}{\cos^2\varphi} = \rho}$



$$D' : \begin{cases} 0 \le \varphi \le \frac{\pi}{4} \\ \dfrac{a\sin\varphi}{\cos^2\varphi} \le \rho \le \sqrt{2}a \end{cases}$$

6.w.

$$\iint_{D'} \frac{\cancel{\rho}\cos\varphi}{\cancel{\rho^2}} \cdot \cancel{\rho}\, d\varphi d\rho = \iint_{D'} \cos\varphi\, d\varphi d\rho =$$

$$= \int_0^{\frac{\pi}{4}} d\varphi \int_{\frac{a\sin\varphi}{\cos^2\varphi}}^{\sqrt{2}a} \cos\varphi\, d\rho =$$

$$= \int_0^{\frac{\pi}{4}} \left(\sqrt{2}a - \frac{a\sin\varphi}{\cos^{\cancel{2}}\varphi}\right) \cancel{\cos\varphi}\, d\varphi =$$

$$= \sqrt{2}a \int_0^{\frac{\pi}{4}} \cos\varphi\, d\varphi - a \int_0^{\frac{\pi}{4}} \operatorname{tg}\varphi\, d\varphi =$$

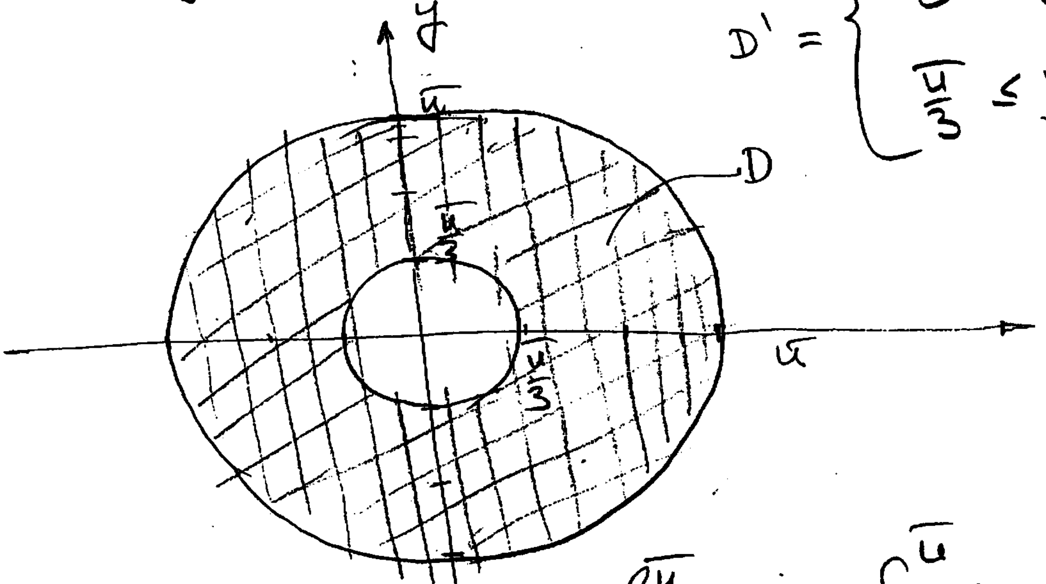
$$= \sqrt{2}a\left(\sin\frac{\pi}{4} - \cancel{\sin 0}^{\,0}\right) + a \ln|\cos\varphi| \Big|_0^{\pi/4} =$$

$$= \sqrt{2}a \cdot \frac{1}{\sqrt{2}} + a\left(\ln\left(\cancel{\cos\frac{\pi}{4}}^{\frac{1}{\sqrt{2}}}\right) - \ln\left(\cancel{\cos 0}^{\,0}\right)\right) =$$

$$= a + a\ln\left(\frac{1}{\sqrt{2}}\right) = \underline{\underline{a\left(1 + \ln\left(\frac{1}{\sqrt{2}}\right)\right)}}$$

9. $\iint\limits_D \frac{\sin\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,dx\,dy$, $D=\{(x,y) \mid \frac{\pi^2}{9} \le x^2+y^2 \le \pi^2\}$

$D' = \begin{cases} 0 \le \varphi \le 2\pi \\ \frac{\pi}{3} \le \rho \le \pi \end{cases}$

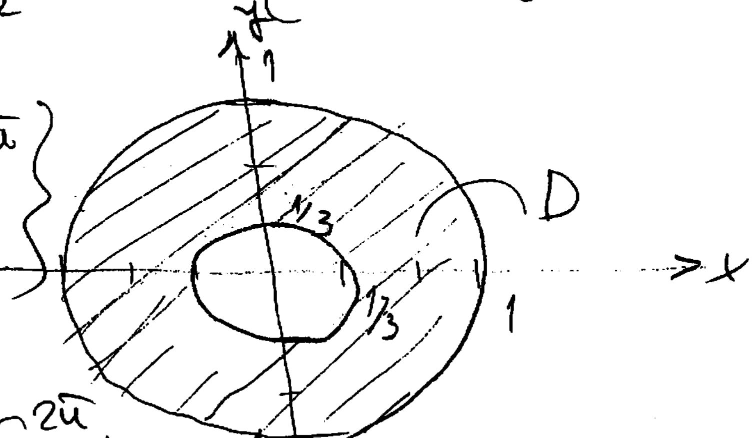


$\iint\limits_{D'} \frac{\sin\rho}{\rho}\cdot\rho\, d\varphi\, d\rho = \int_0^{2\pi} \sin\varphi\, d\varphi \int_{\pi/3}^{\pi} d\rho =$

10. $\iint\limits_D \frac{dx\,dy}{\sqrt{x^2+y^2}}$, $D=\{(x,y) \mid \frac{1}{9} \le x^2+y^2 \le 1\}$

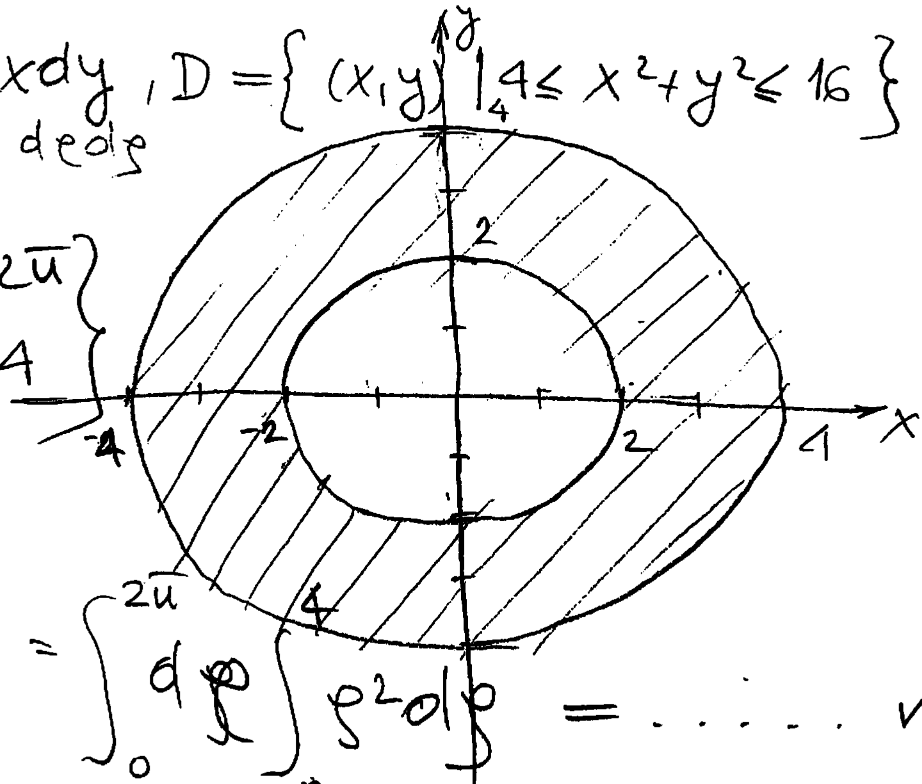
ρ

$D' = \begin{cases} 0 \le \varphi \le 2\pi \\ \frac{1}{3} \le \rho \le 1 \end{cases}$



$\iint\limits_{D'} d\varphi\, d\rho = \int_0^{2\pi} d\varphi \int_{1/3}^{1} d\rho = \ldots\ldots$

11.) $\iint_D \underbrace{\sqrt{x^2+y^2}}_{\varrho}\,\underbrace{dx\,dy}_{\varrho\, d\varphi\, d\varrho}$, $D=\{(x,y) \mid 4\le x^2+y^2\le 16\}$



$$D' = \left\{ \begin{array}{l} 0 \le \varphi \le 2\pi \\ 2 \le \varrho \le 4 \end{array} \right\}$$

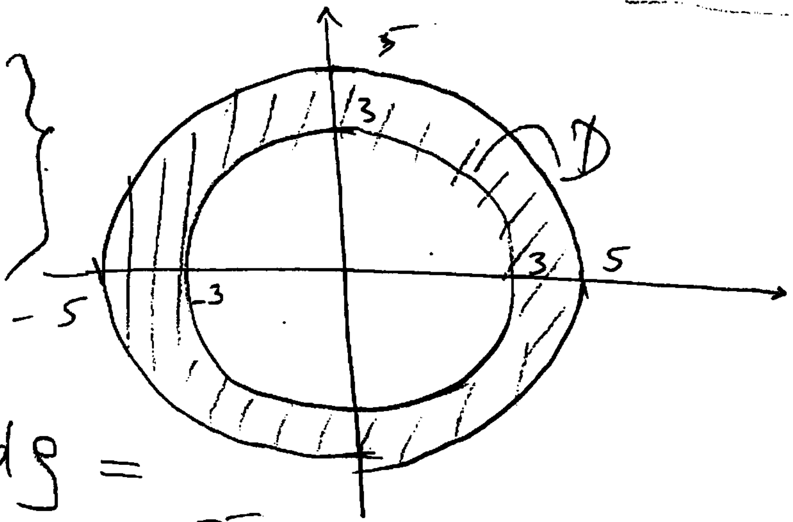
$$\iint_{D'} \varrho^2\, d\varphi\, d\varrho = \int_0^{2\pi} d\varphi \int_2^4 \varrho^2\, d\varrho = \ldots\ldots \checkmark$$

12.) $\iint_D \sqrt{x^2+y^2-9}\,dx\,dy$, $D=\{(x,y) \mid 9\le x^2+y^2\le 25\}$

$\varrho^2\cos^2\varphi+\varrho^2\sin^2\varphi-9$

ϱ^2-9

$$D' = \left\{ \begin{array}{l} 3 \le \varrho \le 5 \\ 0 \le \varphi \le 2\pi \end{array} \right\}$$



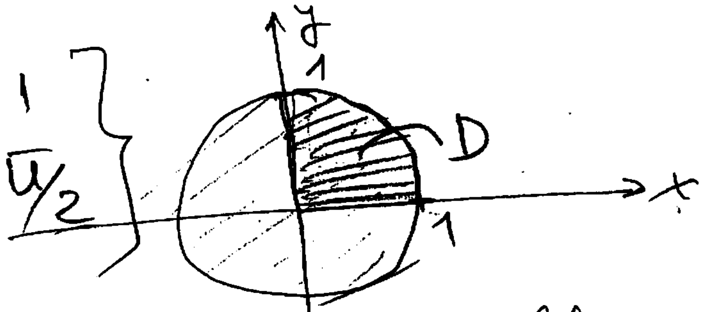
$$\iint_{D'} \sqrt{\varrho^2-9}\cdot \varrho\, d\varphi\, d\varrho = \int_0^{2\pi} d\varphi \int_3^5 \varrho\sqrt{\varrho^2-9}\, d\varrho = \int_0^{2\pi} d\varphi \int_3^5 \tfrac{1}{2}\sqrt{t}\, dt = \ldots$$

$$\left\{ \begin{array}{l} \varrho^2-9=t \\ 2\varrho\, d\varrho = dt \\ \varrho\, d\varrho = \tfrac{1}{2}dt \end{array} \right\}$$

$\ldots$ ✓ ✓

(13.) $\iint\limits_D \frac{dx\,dy}{\sqrt{1-x^2-y^2}}$, $D=\{(x,y)\mid x^2+y^2\leq 1 \wedge x\geq 0 \wedge y\geq 0\}$

$D' = \left\{\begin{matrix} 0\leq\rho\leq 1 \\ 0\leq\varphi\leq\pi/2\end{matrix}\right\}$



$$\iint\limits_{D'} \frac{\rho\cdot d\rho\, d\varphi}{\sqrt{1-\rho^2\cos^2\varphi-\rho^2\sin^2\varphi}} = \iint\limits_{D'} \frac{\rho\, d\varphi\, d\rho}{\sqrt{1-\rho^2}} =$$

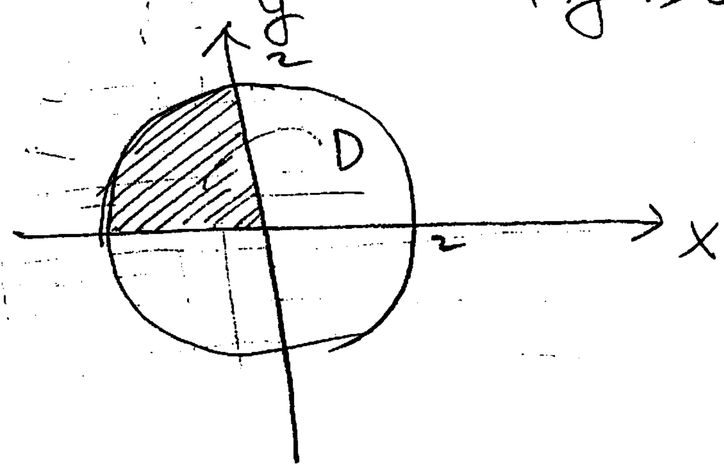
$$= \int_0^{\pi/2} d\varphi \int_0^1 \frac{\rho\, d\rho}{\sqrt{1-\rho^2}} = \left\{\begin{matrix} 1-\rho^2=t \\ -2\rho\, d\rho = dt \\ \rho\, d\rho = -\frac{1}{2}dt\end{matrix}\right\} =$$

$$= -\frac{1}{2}\int_0^{\pi/2} d\varphi \int_0^1 \frac{dt}{\sqrt{t}} = \ldots\ldots \checkmark$$

вратисе на кра[illegible]

(14.) $\iint\limits_D x\,dx\,dy$, $D=\{(x,y)\mid x^2+y^2=4 \wedge x\leq 0 \wedge y\geq 0\}$

$D' = \left\{\begin{matrix} 0\leq\rho\leq 2 \\ \frac{\pi}{2}\leq\varphi\leq\pi\end{matrix}\right\}$

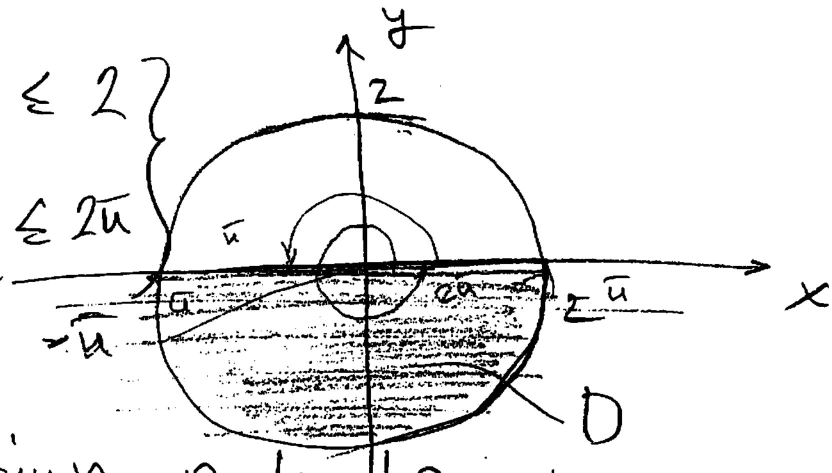


$$\iint\limits_{D'} \rho^2\cos\varphi\, d\varphi\, d\rho =$$

$$= \int_0^2 \rho^2 d\rho \int_{\frac{\pi}{2}}^{\pi} \cos\varphi\, d\varphi = \ldots\ldots \checkmark$$

15. $\iint_D xy\,dx\,dy$, $D=\{(x,y)\mid x^2+y^2\le 4 \wedge y\le 0\}$

$$D'=\left\{\begin{array}{l} 0\le\rho\le 2 \\ \pi\le\varphi\le 2\pi \end{array}\right\}$$



$$\iint_{D'} |\rho\cos\varphi\cdot\rho\sin\varphi\cdot\rho\,d\varphi\,d\rho =$$

$$=\iint_{D'}\rho^3\underbrace{\cos\varphi\sin\varphi}_{\varphi+\varphi}\,d\varphi\,d\rho = \frac{1}{2}\iint_{D'}|\rho^3\sin 2\varphi\,d\varphi$$

$$=\int_0^2\rho^3\,d\rho\int_{\pi}^{2\pi}\frac{1}{2}(\cancel{\sin 0}^{\,0}+\sin 2\varphi)\,d\varphi =$$

$$=\frac{1}{2}\int_0^2\rho^3\,d\rho\int_{\pi}^{2\pi}\sin 2\varphi\,d\varphi = \left\{\begin{array}{l} 2\varphi=t \\ 2d\varphi=dt \\ d\varphi=\frac{1}{2}dt \end{array}\right\}, =\frac{1}{2}\int\sin t\,dt$$

$$=\frac{1}{2}\int_0^2\rho^3\,d\rho\left[-\frac{1}{2}\cos 2\varphi\Big|_{\pi}^{2\pi}\right] = -\frac{1}{4}\int_0^2\rho^3(\cancel{\cos 4\pi-\cos 2\pi})^{0}\,d\rho$$

$$=-\frac{1}{4}\int_0^2\rho^3\cdot 0\,d\rho = 0$$

(16.) $\iint_D |xy|\,dx\,dy$, $D = \{(x,y) \mid x^2+y^2 \le 4 \wedge y \le 0\}$

$$D' = \begin{cases} 0 \le \rho \le 2 \\ \pi \le \varphi \le 2\pi \end{cases}$$

$|\frac{1}{2}\rho^2 \sin 2\varphi|$

$$\iint_{D'} |\rho^2 \cos\varphi \sin\varphi|\, \rho\, d\rho\, d\varphi =$$

$$= \iint_{D'} \sqrt{\frac{1}{2}\rho^2 \sin 2\rho}\ \rho\, d\rho\, d\varphi =$$

$$= \frac{1}{\sqrt{2}} \iint_{D'} \rho^2 \sin 2\varphi\, d\varphi\, d\rho = \frac{1}{\sqrt{2}} \int_0^2 \rho^2 d\rho \int_{\pi}^{2\pi} \sin 2\varphi\, d\varphi$$

$$-\frac{1}{2}\cos 2\varphi \Big|_{\pi}^{2\pi}$$

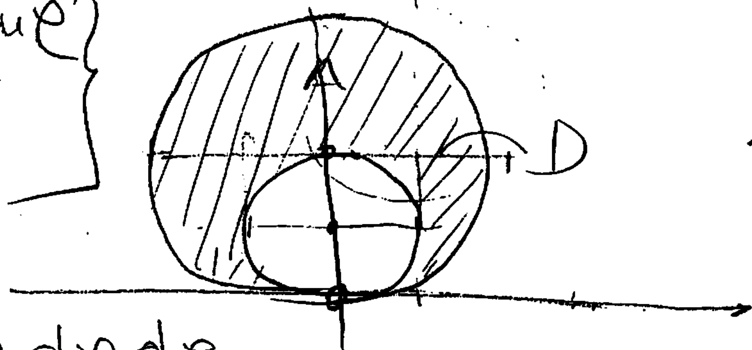
$$-\frac{1}{2}(\cos 4\pi - \cos 2\pi)$$

(18.) $\iint_D xy^2\,dx\,dy$, D областа ограничена кружницата

$\boxed{x^2+(y-1)^2=1}$ $x^2+y^2-2y+1-1=0$, $x^2+y^2=2y$

$x^2+y^2=4y \rightarrow$ $x^2+(y-2)^2=4$

$$D' = \begin{cases} 2\sin\varphi \le \rho \le 4\sin\varphi \\ 0 \le \varphi \le \pi \end{cases}$$



$$\iint_D \rho\cos\varphi\, \rho^2 \sin^2\varphi \cdot \rho\, d\varphi\, d\rho =$$

$$= \iint_D \rho^4 \cos\varphi \sin^2\varphi\, d\varphi\, d\rho = \frac{1}{2}\iint_D \rho^4 \sin 2\varphi \cdot \sin\varphi\, d\varphi\, d\rho$$

$$= \frac{1}{2}\iint_D \rho^4 \left(\frac{1}{2}\cdot(\cos\varphi - \cos 3\varphi)\right) d\varphi\, d\rho =$$

$$= \frac{1}{4}\int_0^{\pi}(\cos\varphi - \cos 3\varphi)\,d\varphi \int_{2\sin\varphi}^{4\sin\varphi} \rho^4\,d\rho =$$

$$= \frac{1}{4}\int_0^{\pi}(\cos\varphi - \cos 3\varphi)\,d\varphi \left(\frac{\rho^5}{5}\Big|_{2\sin\varphi}^{4\sin\varphi}\right) =$$

$$= \frac{1}{4}\int_0^{\pi}\frac{1}{5}(\underbrace{4^5\sin^5\varphi}_{1024\sin^5\varphi} - \underbrace{2^5\sin^5\varphi}_{32\sin^5\varphi})(\cos\varphi - \cos 3\varphi)\,d\varphi =$$

$$= \frac{1}{20}\int_0^{\pi} 992\sin^5\varphi\,(\cos\varphi - \cos 3\varphi)\,d\varphi =$$

$$= \frac{992}{20}\int_0^{\pi}(\sin^5\varphi\cos\varphi - \sin^5\varphi\cos 3\varphi)\,d\varphi =$$

$$= \frac{992}{20}\boxed{\int_0^{\pi}\sin^5\varphi\cos\varphi\,d\varphi} \frac{992}{20}\int_0^{\pi}\sin^5\varphi\cos 3\varphi\,d\varphi =$$

$$\left\{\begin{matrix}\sin\varphi = t \\ \cos\varphi\,d\varphi = dt\end{matrix}\right\}$$

$$\int_0^{\pi} t^5\,dt = \frac{t^6}{6}\Big|_0^{\pi} = \frac{\pi^6}{6}$$

$$\int_0^{\pi}\sin^5\varphi \overbrace{\cos 3\varphi}^{\sin\varphi\cos 3\varphi}\,d\varphi = \int_0^{\pi}\sin^5\varphi\,(4\cos^3\varphi - 3\cos\varphi)$$

$$= 4\int_0^{\pi}\sin^5\varphi\cos^3\varphi - 3\boxed{\int_0^{\pi}\sin^5\varphi\cos\varphi}$$

$$\to \frac{\pi^6}{6}$$

$*$

$$\int_0^{\pi} \sin^5\varphi\cos^3\varphi\,d\varphi = \left\{\begin{matrix}\sin\varphi = t\\ \cos\varphi\,d\varphi = dt\end{matrix}\right\} =$$

$$= \int_0^{\pi} t^5(1-t^2)\,dt = \int_0^{\pi}(t^5-t^7)\,dt =$$

$$= \frac{t^6}{6}\Big|_0^{\pi} - \frac{t^8}{8}\Big|_0^{\pi} = \frac{\sin^6\varphi}{6}\Big|_0^{\pi} - \frac{\sin^8\varphi}{8}\Big|_0^{\pi}$$

$$= \frac{992}{20}\cdot\frac{\pi^6}{6} - \frac{992}{20}\cdot\left(\frac{4}{6}\sin^6\varphi\Big|_0^{\pi} - \frac{4}{8}\sin^8\varphi\Big|_0^{\pi} - 3\frac{\pi^6}{6}\right) =$$

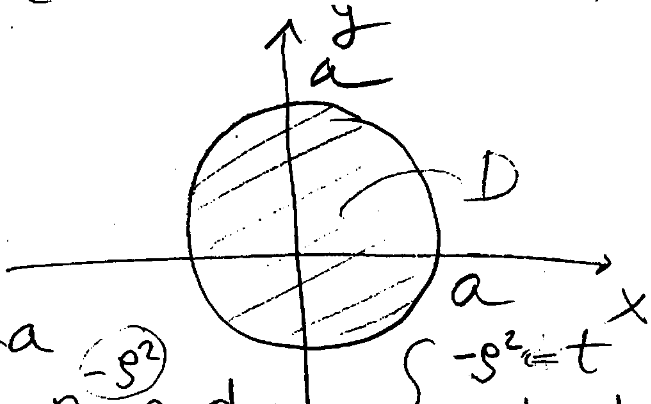
$$= \frac{992}{20}\cdot\frac{\pi^6}{6} - \cancel{\frac{992}{20}\cdot\frac{4}{6}\left(\sin^6\pi - \sin^6 0\right)} - \cancel{\frac{992}{20}\cdot\frac{4}{8}\left(\sin^8\pi - \sin^8 0\right)} -$$

$$+ \frac{992}{20}\cdot\frac{3\pi^6}{6} = \frac{992\pi^6}{20\cdot 6}(1+3) = \frac{992}{5\cdot 6}\pi^6 = \boxed{\frac{992}{30}\pi^6}$$

19. $\iint_D e^{-x^2-y^2}\,dx\,dy$, $D = \{(x,y) \mid x^2+y^2 \le a^2\}$

$-(x^2+y^2)$

$$D' = \left\{\begin{matrix}0\le\rho\le a\\ 0\le\varphi\le 2\pi\end{matrix}\right\}$$



$$\iint_{D'} e^{-\rho^2}\rho\,d\rho\,d\varphi = \int_0^{2\pi}d\varphi\int_0^a e^{-\rho^2}\rho\,d\rho = \left\{\begin{matrix}-\rho^2 = t\\ -2\rho\,d\rho = dt\\ \rho\,d\rho = -\frac{1}{2}dt\end{matrix}\right\}$$

$$= \int_0^{2\pi}d\varphi\int_0^a e^t\left(-\frac{1}{2}\right)dt = -\frac{1}{2}\int_0^{2\pi} e^{-\rho^2}\Big|_0^a\,d\varphi =$$

$$= -\frac{1}{2}e^{-a^2}\int_0^{2\pi}d\varphi = -\frac{1}{2}e^{-a^2}\cdot\varphi\Big|_0^{2\pi} = -\pi e^{-a^2}$$

(20) $\iint_D x\sqrt{x^2+y^2}\,dx\,dy$, $D=\{(x,y) \mid (x^2+y^2)^2 \le x^2-y^2 \wedge x \ge 0$

$\boxed{(x^2+y^2)^2 \le x^2-y^2} \to (x-y)(x+y)$

$x^4+2x^2y^2+y^4-x^2+y^2 \le 0$?

(21) $\int_0^a dy \int_{\sqrt{ay-y^2}}^{\sqrt{a^2-y^2}} \frac{dx}{\sqrt{a^2-x^2-y^2}}$

$x=\sqrt{a^2-y^2}$

$\boxed{x^2+y^2=a^2}$

$\rho^2=a^2$

$\boxed{\rho=a}$

$\begin{cases} 0 \le y \le a \\ \sqrt{ay-y^2} \le x \le \sqrt{a^2-y^2} \end{cases}$

$a\cancel{\rho}\sin\varphi = \rho^{\cancel{2}}$

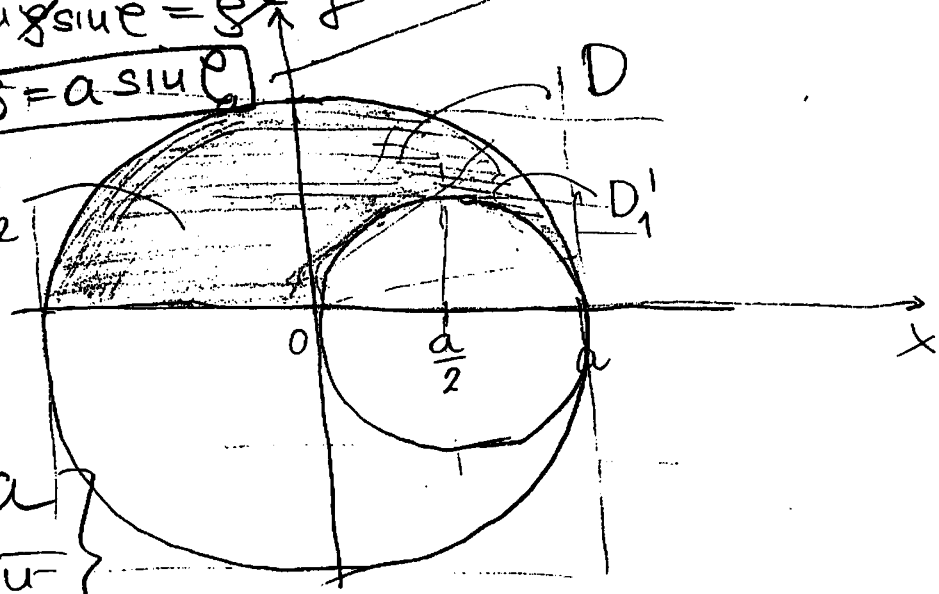
$\boxed{\rho = a\sin\varphi}$

$ay-y^2=x^2$

$\boxed{ay=x^2+y^2}$

$x^2+y^2-ay=0$

$\boxed{x^2+\left(y-\frac{a}{2}\right)^2=\frac{a^2}{4}}$



$D_1' = \begin{cases} a\sin\varphi \le \rho \le a \\ 0 \le \varphi \le \frac{\pi}{2} \end{cases}$

$D_2' = \begin{cases} 0 \le \rho \le a \\ \frac{\pi}{2} \le \varphi \le \pi \end{cases}$